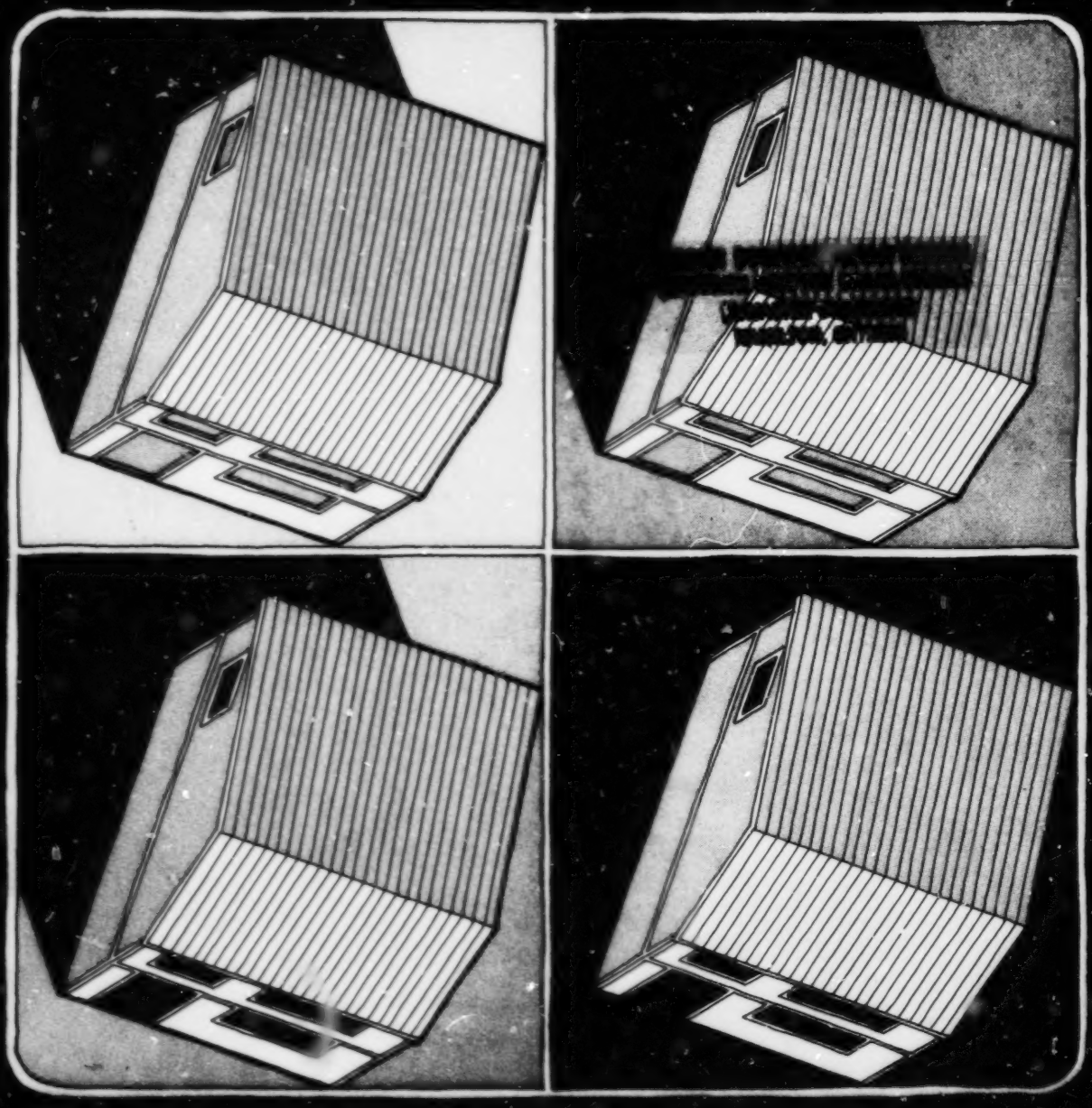
HANDBOOK FOR ENERGY EFFICIENT RESIDENTIAL SUBDIVISION PLANNING

Shadows for Site Planning



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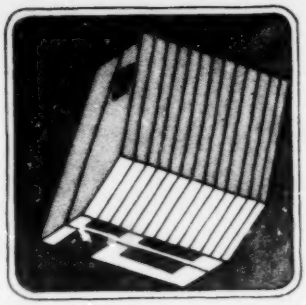
HANDBOOK FOR ENERGY EFFICIENT RESIDENTIAL SUBDIVISION PLANNING



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Ministry of Municipal Affairs
and Housing

Project Planning Branch

Handbook for Energy Efficient
Residential
Subdivision Planning

Part 2:
Shadows for Site Planning

Ministry of Municipal Affairs and Housing

Project Planning Branch

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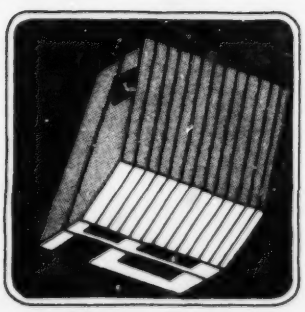
PART 2

Shadows for Site Planning



In January 1981, the Ministry of Housing issued a report on *Residential Site Design and Energy Conservation*. This basic study examined the possibilities of saving energy from conventional sources used for space heating. Houses were located within a subdivision with the main window areas facing south to utilize the radiation from the sun. This *Handbook for Energy Efficient Residential Subdivision Planning* is a consequence of the basic study.

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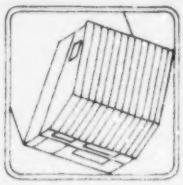

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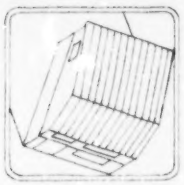
Foreword

The *Handbook for Energy Efficient Residential Subdivision Planning* provides a comprehensive design guide for municipal planners and land developers. It shows how to design residential subdivisions that can reduce space heating needs with only marginal additional development costs by taking account of local solar and wind conditions.

The Handbook consists of a number of parts that are published separately. Each part deals with some aspect of subdivision design relevant to saving energy. Part 1, the Overview, describes each part and the relationship to the total subject.

This document serves to examine the effects of shadow projections on site design, and to identify the methods of calculating these shadows.

Introduction



Purpose

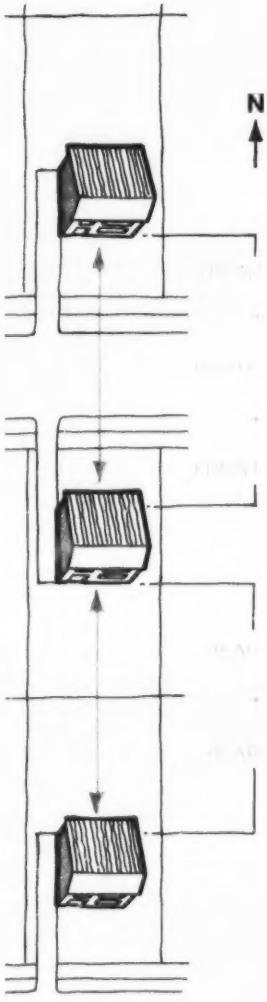
This part of the *Handbook for Energy Efficient Residential Subdivision Planning* deals with the problem of the shading of one building by another during critical hours of the heating season. Site planning for solar access is achieved by knowing the shadow pattern — which is the composite shape of a shadow cast by an object over a given period of time.

Shadows for Site Planning has two objectives:

- It demonstrates the effects on energy conscious subdivision design of using shadow length calculations.
- It examines some methods for the calculation of those shadows.

The importance of evaluating the influence of shadows on site design is apparent from the fact that energy from the sun can account for a significant percentage of a dwelling's heating needs. One of the prerequisites for using this solar radiation are houses in a subdivision sited and designed so that they do not cast shadows on nearby dwellings during a particular critical time span during a day known as the design period. A window in total shade does not admit direct sunlight into a house. Therefore, the knowledge of shadow projections of dwellings aids a subdivider in making decisions about such matters as lot depth and street orientation.

Depending upon the height of the house, the latitude of the site and the topography of the land, it is possible to determine whether houses in a subdivision can have access to direct solar radiation. This is done by comparing shadow lengths with the likely distance between dwellings (comprising either rear yard depths or front yard setbacks plus right -of -way).



Likely distance between dwellings used to determine possibility of solar access.

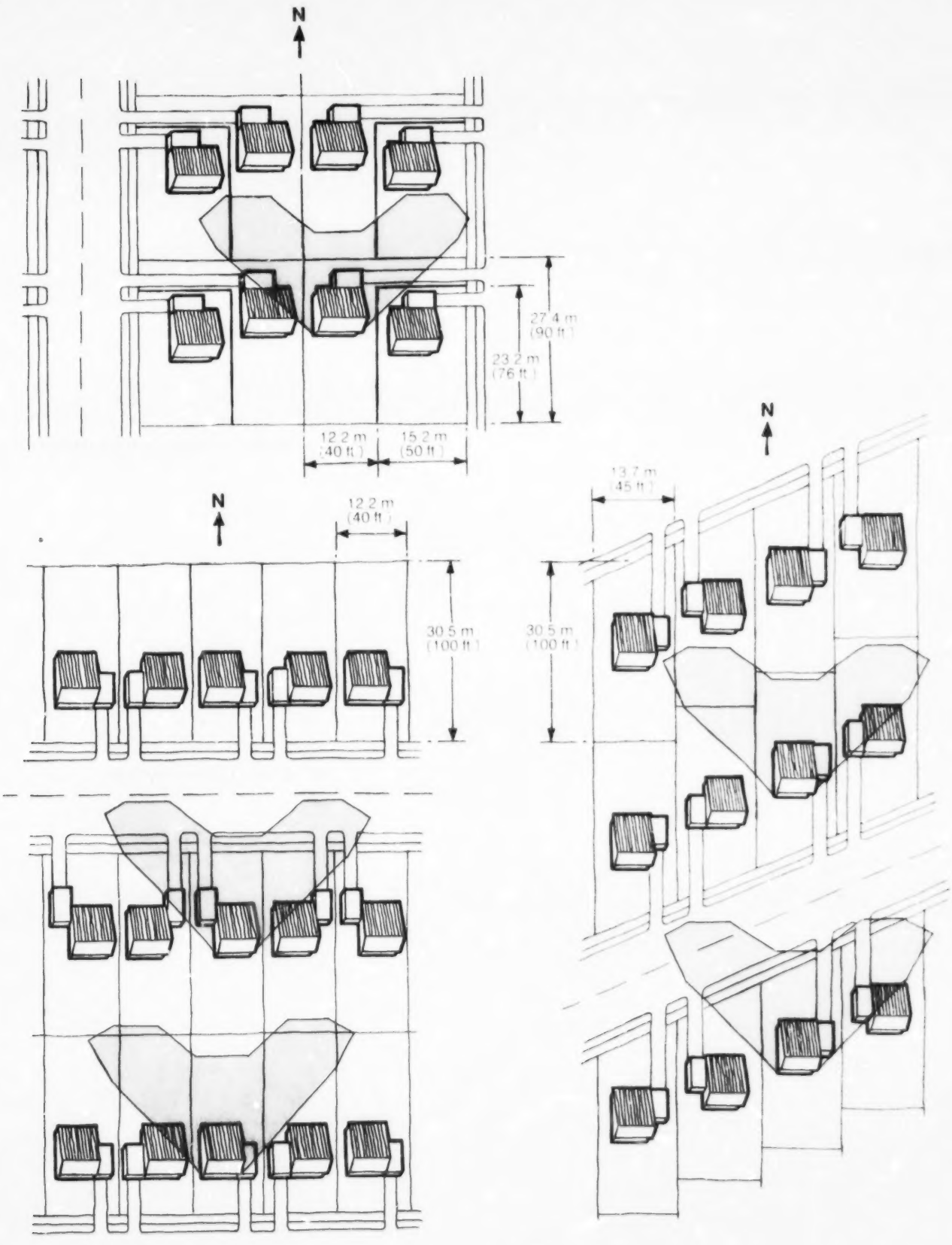
At any particular site, there are four possible ways the material in this document can be used:

- Determining the shadow pattern of a house.
- Determining the shadow patterns for several typical dwellings in various orientations — these can serve as references during the design of streets and lots.
- Providing the basis for the development of solar envelope zoning.
- Providing the basis for the development of performance standards for protecting solar access.

Shadows, housing unit yield and development costs

There needs to be no significant difference in the housing unit yield on a more or less level site for most of the urbanized areas of the Province as a result of applying the shadow patterns discussed in this document. For example, the drawings of the sample situations at the latitude of Ottawa show that the shadow patterns fall well within lots of 30.5 metres (100 feet) in depth.

Studies have shown that using shadow patterns as a criterion for development does not compromise other development objectives, such as servicing costs and lot yield. This has been demonstrated in two previous reports, *Residential Site Design and Energy Conservation* and *Residential Site Design Alternatives in Small Communities*, and is to be further detailed in a forthcoming part to this Handbook.



Representative shadow patterns for dwellings on three street alignments at Ottawa latitude.

The effects of shadow calculations on site planning

Shadow patterns

Shadow patterns are related to:

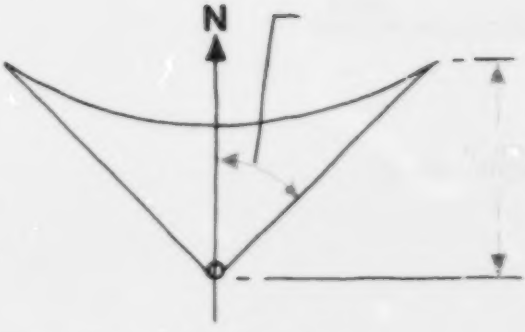
- The building (dimensions and orientation).
- The site (slope gradient and direction).
- The location of the sun in the sky (solar azimuth and altitude), which varies with the site latitude, day of the year, and time of the day.
- The tree.

There are two basic measurements in a daily shadow pattern:

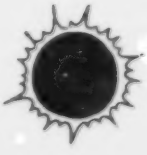
- The north shadow projection (a linear measurement) is the distance from the edge of the shading object to the most northerly point of the shadow shape.
- The lateral shadow projection (an angular measurement) is determined by the solar azimuth angle, and is the angle outside which neighbouring dwellings to the west and the east should be sited.

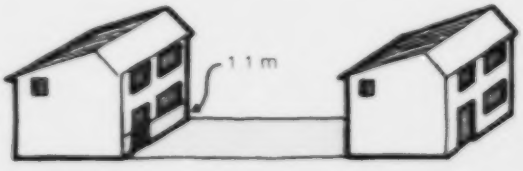
The first of these measurements is the one usually taken as the critical distance between buildings to avoid shading.

In order for a south-facing wall to have solar access, it is only necessary for the window area to be shadow free. This means that the usual 1.1 m (3 ft. 8 in.) of space between the ground and the bottom of the ground floor window may be shadowed with virtually no reduction in potential solar gains.



The two basic measurements of a shadow pattern.





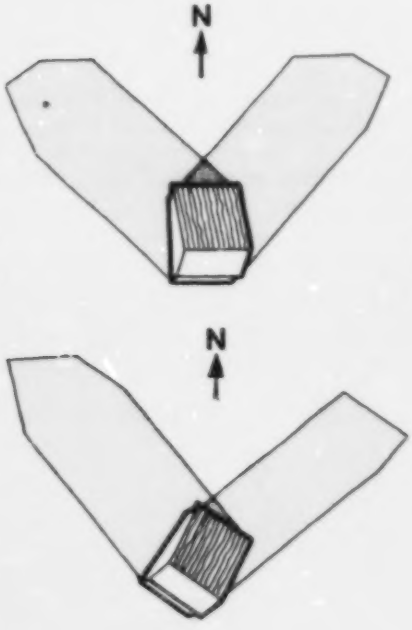
Shadow may reach to 1.1 m of space below window without reducing solar potential.

The building

In the usual residential subdivision the height of the house is the critical shading feature.

Determining building heights in order to calculate shadow patterns may be done in one of three ways:

- Use the height as shown in the actual house design.



House orientation affects shadow pattern.

- Use the maximum allowable dwelling height as provided for in local zoning by-laws. However, using prescribed maximums as the design height may, in some cases, yield unusually long shadows. This results in dwellings spaced especially far apart in order to ensure solar access. Often, these zoned limits can exceed those actually used.

- Assume what could normally be expected, considering surrounding development and marketing trends. The resultant spacing of dwellings is more likely to reflect what is eventually built.

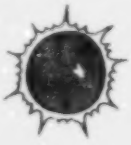
Orientation of a dwelling also affects the shadow shape. When a dwelling is turned off a due south orientation, the morning and afternoon portions of the shadow pattern become asymmetrical.

The site

There are two relevant factors — slope and latitude.

Dwellings sited on a north-facing slope cast longer shadows (and require greater spacing between dwellings) than similar dwellings on a south slope. For example, at Southern Ontario la..tudes, when a house has a north shadow projection equal to about three times its height on level ground, this shadow projection is increased to approximately four times on a 10 percent north slope.

The higher the latitude, the greater is the north shadow projection. For example, north shadow projection on January 21 at 42°N (the Windsor area) is only about 65 percent of that at 50°N (the Kenora area).



Direction of slope affects shadow lengths.

The location of the sun in the sky

The sun's position at any given time is an important factor in determining both the shadows cast by a dwelling and the amount of solar radiation received.

December 21 (the winter solstice) is the day of the year when shadow lengths are the longest. If solar access is possible on this date, then access is achieved on all other days of the heating season. However, December 21 is not selected because for most sites in the Province achieving solar access on this date requires dwellings in a subdivision to be spaced unreasonably far apart.

The substitution of January 21 as the day on which to compute shadow lengths allows a more reasonable standard for building separation.

At a low solar altitude (for example, before 9:00 a.m. and after 3:00 p.m. on January 21), the sun's rays travel a longer distance through the atmosphere. A loss of solar radiation results. Thus, shadow pattern determination is generally limited to a six hour period. A shorter time period (for example, 10:00 a.m. to 2:00 p.m.) may be used if trade-offs with other development factors are necessary.

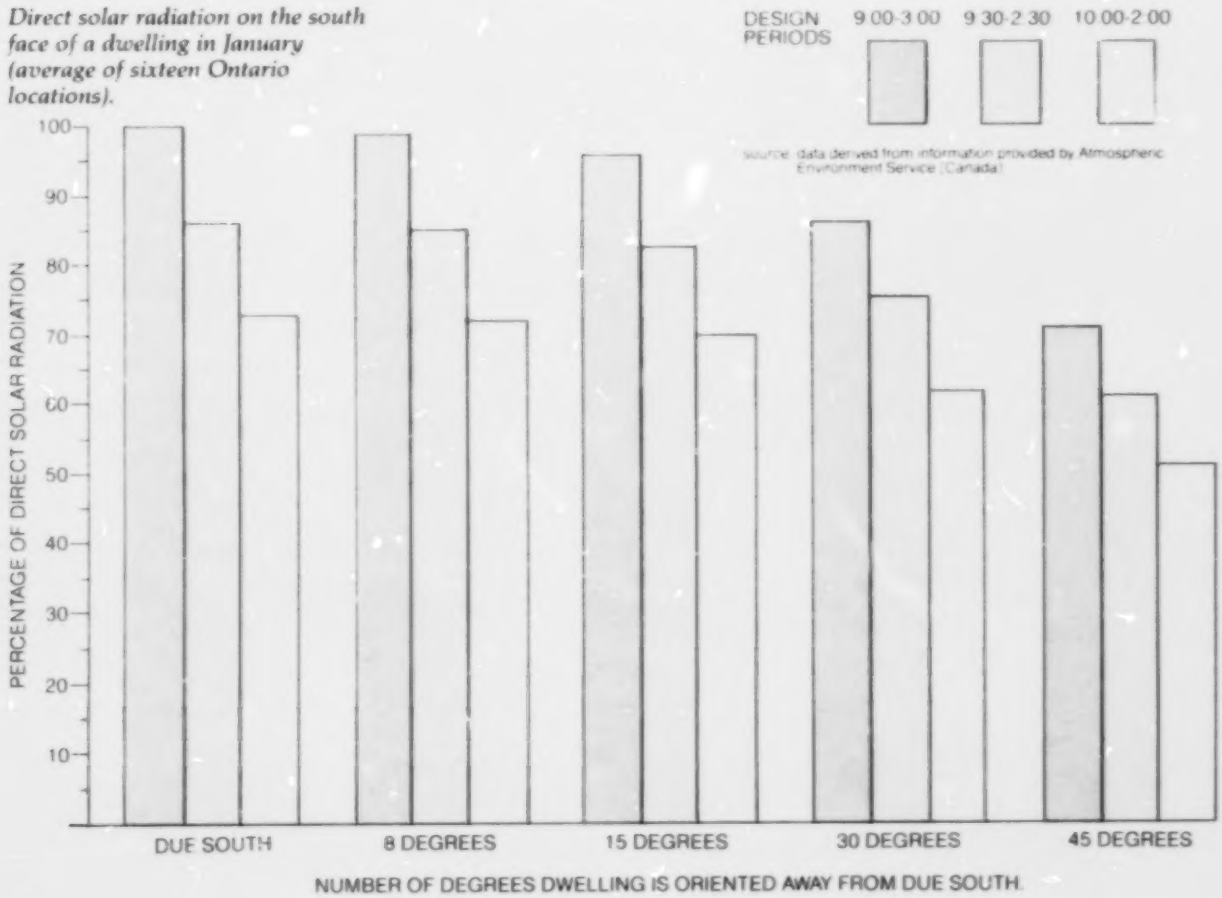
The tree

A tree differs from a building in terms of its shadow projection. Because its shape alters with its age and (with the exception of most evergreens) the season, there is a changing shadow.

The basic consideration in arriving at the degree of interference of solar access to a house by a tree is related to the tree's mature size and whether or not its leaves are shed during the heating season.

The form and relationship of a tree shadow to the heating and cooling of a house are to be dealt with in subsequent part of this Handbook.

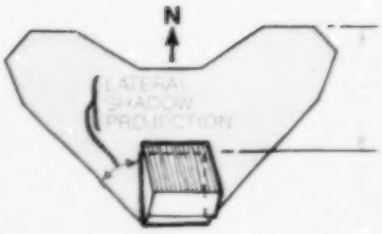
Direct solar radiation on the south face of a dwelling in January (average of sixteen Ontario locations).



The two aspects of shading

Obstructions to solar access because of shading are considered in two situations:

- The north projection of the shadow.
- The lateral projection of the shadow.



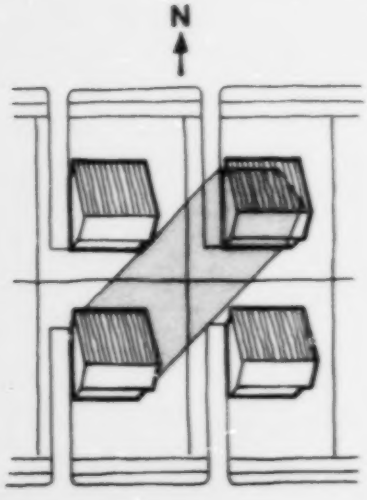
The two basic measurements of a shadow pattern.

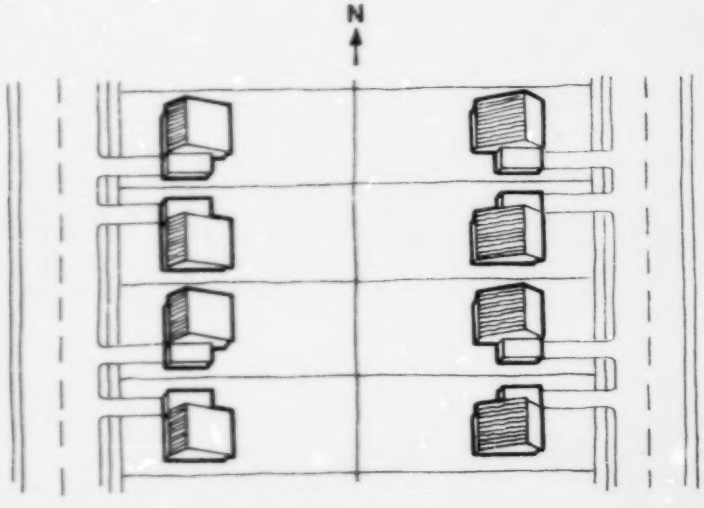
North shadow projection

For a typical two-storey, detached dwelling subdivision development at a location with relatively flat terrain, shadow-free solar access for most dwellings on a north-south axis can usually be accommodated. However, access may be impeded if any one (or combination) of the following circumstances exist:

- Terrain falls to the north.
- Lot depths are unusually short.
- Streets are aligned in a north-south axis.

Possible impediments to solar access.



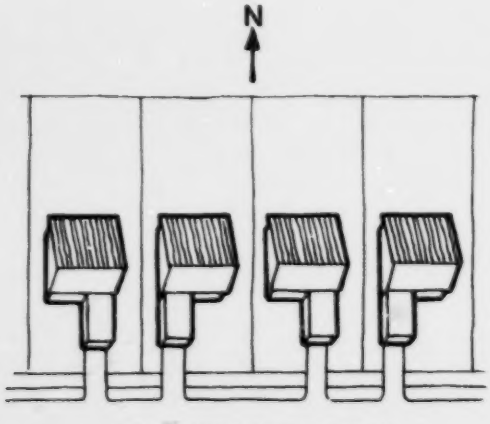


- House types associated with higher densities are located adjacent to and south of ground-related dwellings.
- Garages are sited in front of and adjoining south-facing walls.
- Site is located at a northerly latitude in the Province (where the solar altitude is lower and subsequent shadows are longer).

If solar access were threatened because of one or more of the above factors, a trade-off may be sought. For example, if a proposed house height were found to result in shaded dwellings to the north, an increase in lot depth (and corresponding decrease in lot width to maintain lot area) may have to be considered to offset the increased north shadow projections.

HIGH DENSITY DEVELOPMENT LOCATED TO THE SOUTH OF LOW DENSITY

Possible impediments to solar access.

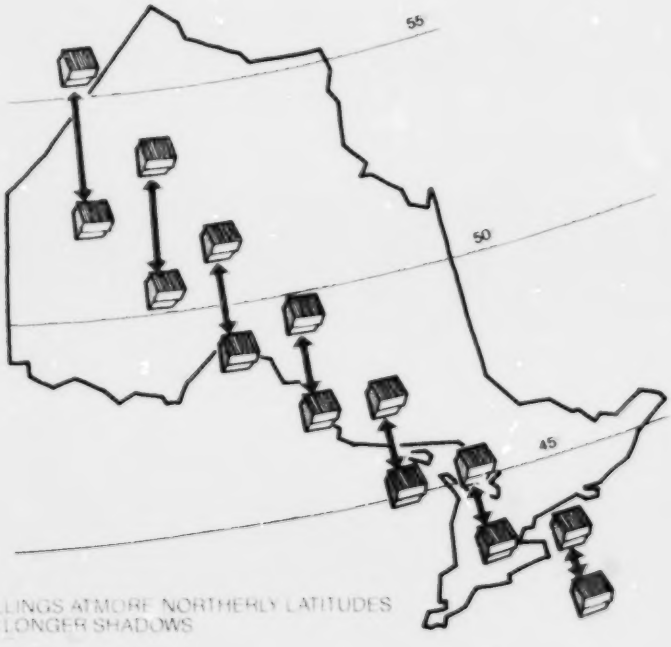


GARAGES INTERFERE WITH SOUTH WALL ACCESS

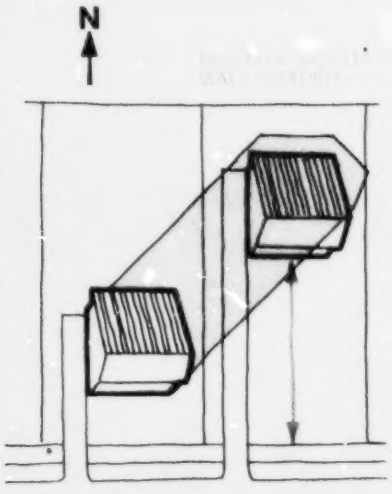
Lateral shadow projection

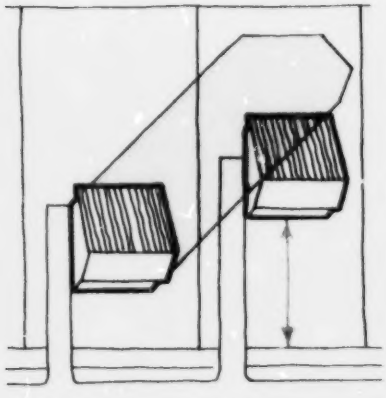
Lateral shadow projections are determined by the azimuth angles of the first morning and last afternoon design hours. At 9:00 a.m. and 3:00 p.m. (which define a typical design period) the sun is at a particular bearing east of south and west of south respectively. Due to the nature of the sun's apparent movement, these angles are virtually the same for corresponding morning and afternoon hours (such as 9:00 a.m. and 3:00 p.m.). For Ontario locations on January 21 this azimuth angle ranges between 41.9° to 43.6° from due south, depending upon the latitude of the site. Azimuth angles for other design times are found in the table of hourly solar altitudes and azimuths (page 14).

Lateral projections of the shadow from one house could result in the limitation of solar access to an adjoining dwelling. Neighbouring houses along the same side of a street are sited according to prescribed front and sideyard setbacks. If, as a result of these setbacks the south wall of a dwelling falls within the lateral shadows defined by the 9:00 a.m. and 3:00 p.m. azimuths (at approximately 45° east and west of the north on a north-south axis) cast by a neighbouring dwelling, that south wall is denied access to the sun during some portion of the design period.



DWELLINGS AT MORE NORTHERLY LATITUDES CAST LONGER SHADOWS

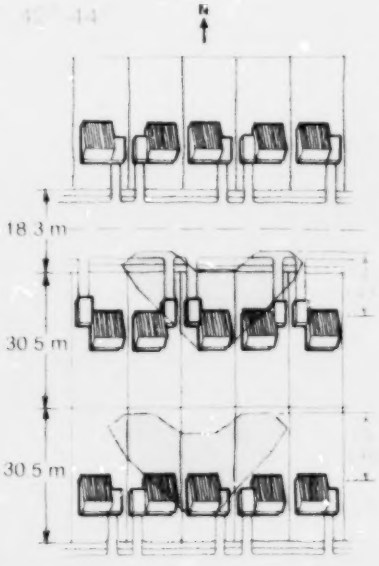




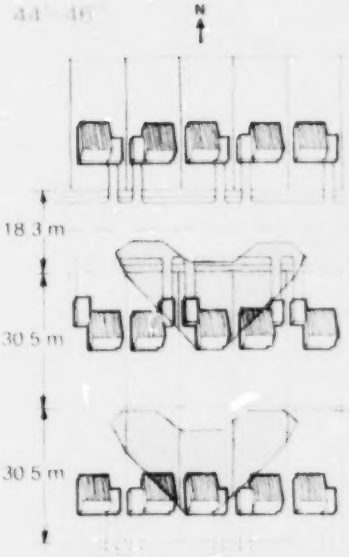
Siting of neighbouring dwellings determines extent of lateral shading interference.

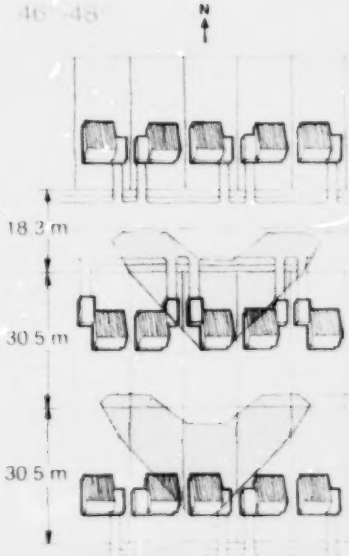
Representative shadow projections

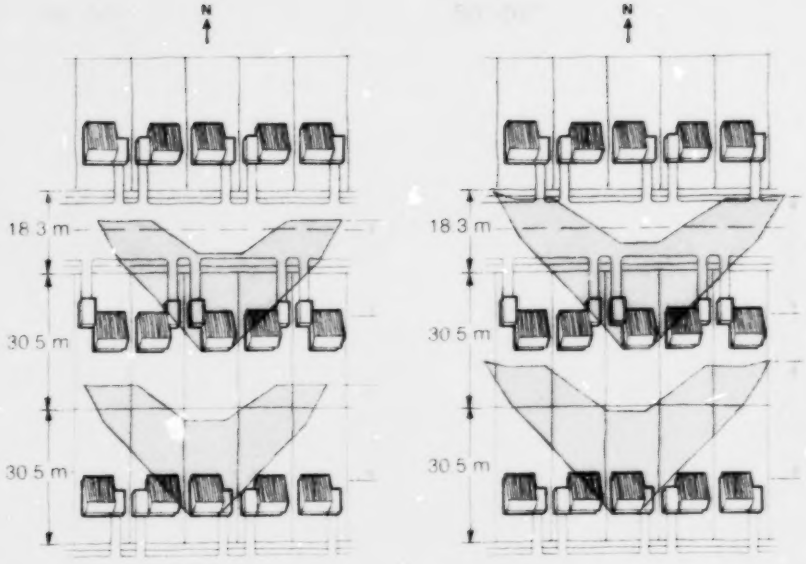
These drawings compare the north shadow projections of a typical two storey dwelling for different latitudes in Ontario. They indicate that shadows cast by this dwelling can usually be accommodated on flat sites using conventional development standards, and having an east-west street orientation. As streets are skewed from this east-west direction, the more difficult it is to assure the sun's access to the south facing glass area of neighbouring dwellings.



Shadow patterns at various latitudes for two-storey detached house.







	ONE STOREY	TWO STOREY	THREE STOREY
42°-44°	7.7 m	14.7 m	23.0 m
44°-46°	8.5 m	16.5 m	25.6 m
46°-48°	9.7 m	18.8 m	29.1 m
48°-50°	11.2 m	21.7 m	33.6 m
50°-52°	13.1 m	25.4 m	39.2 m

This comparison of north shadow projections (on level ground) at different latitudes in Ontario indicates a substantial difference between the situations at 42°-44°N. and 50°-52°N.

Representative sites at various latitudes in Ontario.

Armstrong
Fort Hope
Moosonee
Ogoki
Pickle Lake
Sioux Lookout

Cochrane
Dryden
Fort Frances
Geraldton
Hearst
Iroquois Falls
Kapuskasing
Kenora
Kirkland Lake
Sioux Lookout
Thunder Bay
Timmins

Elliot Lake
Englehart
Espanola
Haileybury
New Liskeard
North Bay
Sault Ste. Marie
Sturgeon Falls
Sudbury

Barrie
Belleville
Brockville
Cornwall
Huntsville
Kingston
Orillia
Ottawa
Owen Sound
Parry Sound
Pembroke
Peterborough

Brantford
Burlington
Cambridge
Chatham
Guelph
Hamilton
Kitchener/Waterloo
London
Niagara Falls
Oshawa
Port Colborne
St. Catharines
St. Thomas
Sarnia
Stratford
Toronto
Welland
Windsor
Woodstock

An outline of the site layout process

An outline of the process to ensure that a maximum number of dwellings in a subdivision receive direct solar radiation for most of the heating season may follow this sequence:

1. Choose a number of representative house types. If the house designs to be used are not available, an approximation of the dimensions and locations of buildings on the lots can be arrived at by examining the zoning bylaw and/or local market trends. These establish representative building heights which can then be used to develop approximate shadow projections.

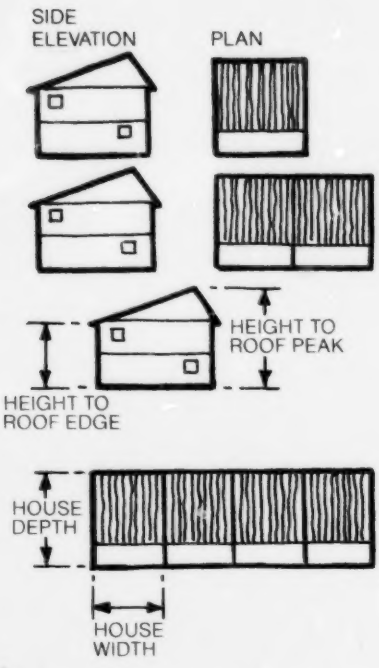
2. Using the methods described in this document, calculate the shadow pattern for each representative case for a particular design period (for example, on January 21, between 9:00 a.m. and 3:00 p.m.).

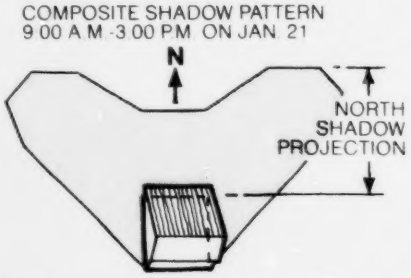
3. With the information derived in 2. above, determine the north shadow projection.

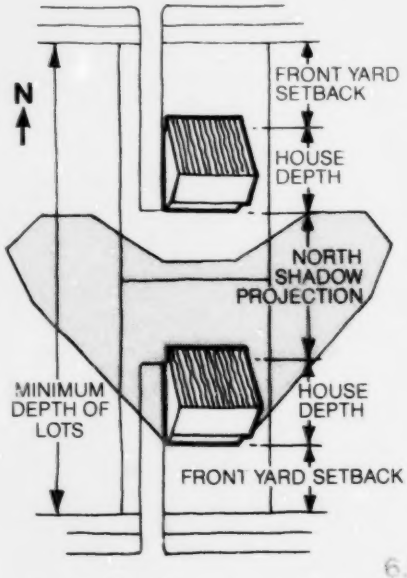
4. Determine front yard setbacks and relate them to local zoning bylaws for:

   - The lot of the shadow casting house.
   - The lots adjacent and to the north.

5. Sum the various components which determine the minimum depth of lots which ensure the possibility for solar access.





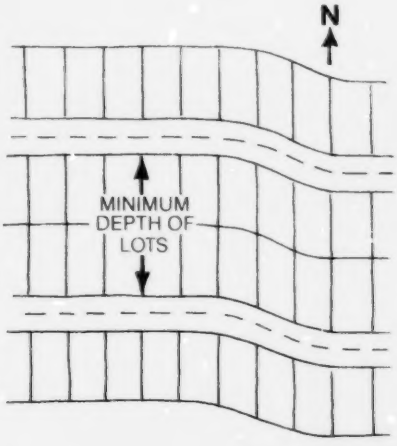


6. Determine the major considerations when laying out a subdivision. Two of these are:

   - The ability of the site to be drained easily.
   - The need to create an east-west street pattern that enables as many major window areas of dwellings (living rooms) as possible to face south. Some deviation from a maximum east-west route may be unavoidable due to topography, avoidance of wind channelling, or environmental and other constraints.

7. Design a street pattern which incorporates, at least, the minimum depth of lots determined in Step 5, and ensure that all dwellings, when they are sited, are outside the lateral shading limits of their neighbours.

The site layout process to conserve energy, and the related development options, are to be examined in detail in a subsequent part of this Handbook.



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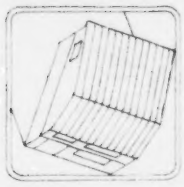
HAMILTON, ONTARIO

Methods of determining shadow patterns



Introduction

Three methods of determining shadow patterns of a dwelling unit are described. Each method varies in complexity and in degree of accuracy, and is suitable for particular types of development situations.

Method A, *Shadows from Geometry* is a relatively simple procedure which requires the drawing of the elevation and plan of the dwelling and the plotting of a few angles to determine shadow lengths. This method is only suited for those dwellings sited on level ground.

Method B, the use of *Shadow Length Multiplier Templates*, is probably the quickest of the three methods as it requires minimal calculation. It can be used for slopes up to 5 percent.

Method C, *Shadow Length Formula*, requires the use of slightly more calculation. However, it provides the highest level of precision and can be applied to any slope situation and design period.

Site specific information related to slope gradient and direction, and solar altitude and azimuth angles are required for completely accurate shadow calculations. Although the information presented here is in generalized form, it does provide a relatively good indication of any shadow problem which may occur in a residential development.

In order to best illustrate the three methods, an example house in a theoretical subdivision is carried through the mechanics of each.

Details of example

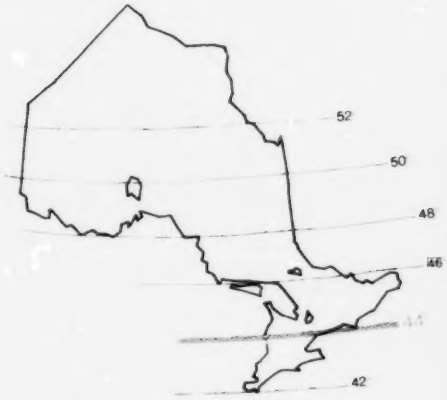
The information required for each of the three methods is indicated as the method is introduced. Although the procedures mostly require the same types of data, certain information, such as the shadow length multiplier templates, is particular to a specific method.

The site

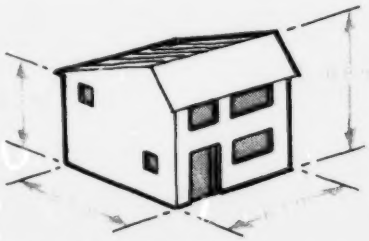
The theoretical subdivision is located at a latitude of 44° North.

The house

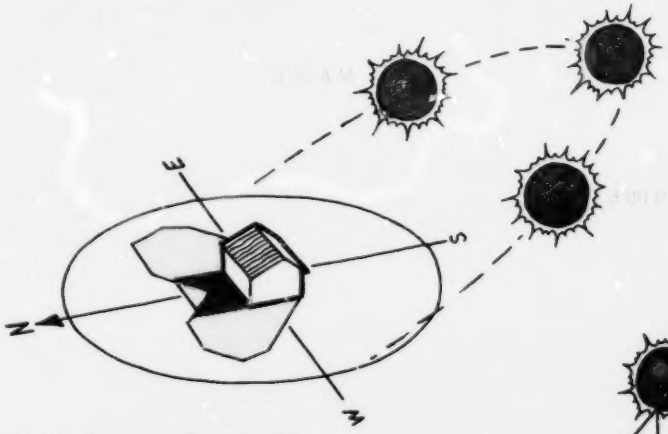
It is a two-storey detached building oriented towards the south, with a roof shape designed to accommodate a future active solar panel.



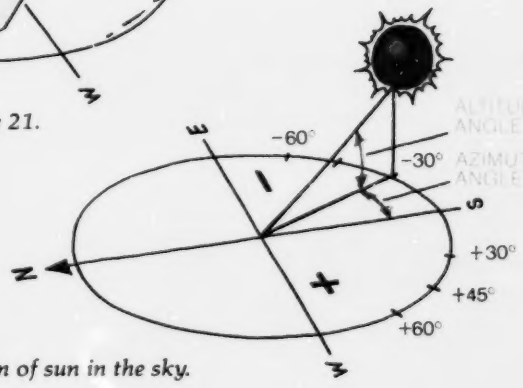
Latitude of site.



Dimensions of dwelling.



Design hours on January 21.



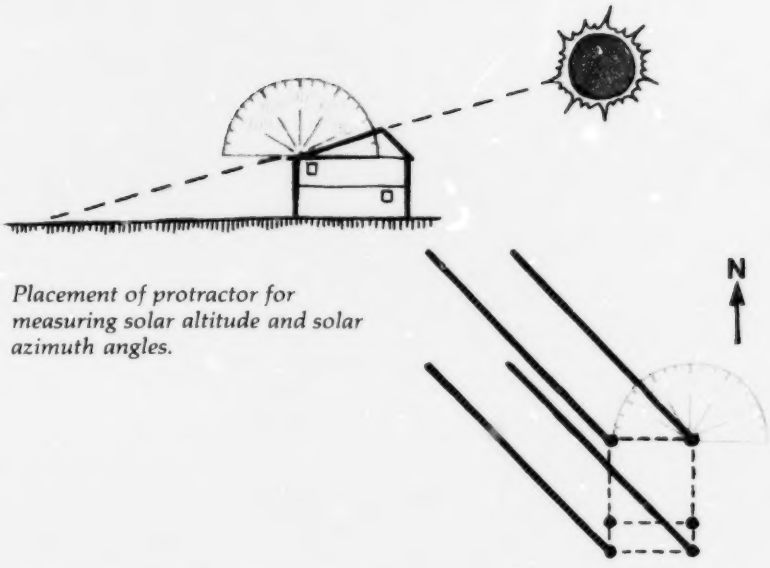
Location of sun in the sky.

The design period

The period between 9:00 a.m. and 3:00 p.m. on January 21 is chosen. This period permits the use of about 85 percent of the available direct solar radiation. At latitude 44°N., the table of hourly solar altitudes and azimuths on this page shows:

	Solar azimuth	Solar altitude
9:00 a.m.	-43.2°	13.9°
noon	0.0°	26.0°
3:00 p.m.	+43.2°	13.9°

Minus azimuth readings are those east of south. Conversely, the positive readings are west of south.



Placement of protractor for measuring solar altitude and solar azimuth angles.

TABLE OF HOURLY SOLAR ALTITUDES AND AZIMUTHS — JANUARY 21

NORTH LATITUDE	TIME OF DAY — azimuth readings: east of south (morning) +azimuth readings: west of south (afternoon)								
	8 00	9 00	10 00	11 00	12 00	1 00	2 00	3 00	4 00
42° altitude	6 8	15 3	22 1	26 4	28 0	26 4	22 1	15 3	6 8
azimuth	-55 1	-43 6	-30 5	-15 7	0 0	+15 7	+30 5	+43 6	+55 1
43° altitude	6 3	14 6	21 2	25 5	27 0	25 5	21 2	14 6	6 3
azimuth	-55 0	-43 4	-30 3	-15 6	0 0	+15 6	+30 3	+43 4	+55 0
44° altitude	5 8	13 9	20 3	24 6	26 0	24 6	20 3	13 9	5 8
azimuth	-54 9	-43 2	-30 1	-15 5	0 0	+15 5	+30 1	+43 2	+54 9
45° altitude	5 2	13 2	19 5	23 6	25 0	23 6	19 5	13 2	5 2
azimuth	-54 8	-43 0	-29 9	-15 4	0 0	+15 4	+29 9	+43 0	+54 8
46° altitude	4 6	12 5	18 7	22 7	24 0	22 7	18 7	12 5	4 6
azimuth	-54 8	-42 9	-29 8	-15 3	0 0	+15 3	+29 8	+42 9	+54 8
47° altitude	4 0	11 7	17 8	21 8	23 0	21 8	17 8	11 7	4 0
azimuth	-54 7	-42 7	-29 6	-15 2	0 0	+15 2	+29 6	+42 7	+54 7
48° altitude	3 4	10 9	16 9	20 8	22 0	20 8	16 9	10 9	3 4
azimuth	-54 6	-42 6	-29 4	-15 1	0 0	+15 1	+29 4	+42 6	+54 6
49° altitude	2 9	10 2	16 0	19 7	21 0	19 7	16 0	10 2	2 9
azimuth	-54 6	-42 5	-29 3	-15 0	0 0	+15 0	+29 3	+42 5	+54 6
50° altitude	2 3	9 5	15 1	18 8	20 0	18 8	15 1	9 5	2 3
azimuth	-54 6	-42 3	-29 2	-14 9	0 0	+14 9	+29 2	+42 3	+54 6
51° altitude	1 7	8 8	14 3	17 8	19 0	17 8	14 3	8 8	1 7
azimuth	-54 5	-42 2	-29 0	-14 8	0 0	+14 8	+29 0	+42 2	+54 5
52° altitude	0 9	8 0	13 4	16 8	18 0	16 8	13 4	8 0	0 9
azimuth	-54 5	-42 2	-28 9	-14 7	0 0	+14 7	+28 9	+42 2	+54 5
53° altitude	–	7 3	12 5	15 8	17 0	15 8	12 5	7 3	–
azimuth	–	-42 1	-28 8	-14 6	0 0	+14 6	+28 8	+42 1	–
54° altitude	–	6 6	11 6	14 8	16 0	14 8	11 6	6 6	–
azimuth	–	-42 0	-28 7	-14 6	0 0	+14 6	+28 7	+42 0	–
55° altitude	–	5 8	10 8	13 9	15 0	13 9	10 8	5 8	–
azimuth	–	-41 9	-28 6	-14 5	0 0	+14 5	+28 6	+41 9	–

source: adapted from National Research Council, Technical Paper No. 243, April 1967

Method A — Shadows from geometry

Description

This method of shadow calculation relies on the measurement of sun angles. It can produce accurate results for sites on level ground only.

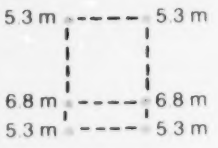
Information required

- Dimensions of the dwelling (in plan and elevation)
- Orientation of the dwelling
- Day of the year
- Time of day (design hour)
- Latitude of the site
- Solar azimuth
- Solar altitude

Process

1. Draw to scale the side elevation of the house.
2. Determine the solar altitude for a particular design hour (in this example it is 9:00 a.m.) according to the latitude of the site as shown in the table of hourly solar altitudes and azimuths.
3. Draw a line at the altitude angle connecting the north edge of the roof with the ground. Measure the distance at the ground level from the north edge of the roof to the point where the altitude angle intersects the ground.
4. Draw a second line at the altitude angle connecting the peak of the roof with the ground. Measure the distance at ground level from the roof peak to the point where the altitude angle intersects the ground.
5. Designate to scale (in plan view) the house as a series of poles representing the heights to the roof edge at the corners of the building and the peak of the roof.

5



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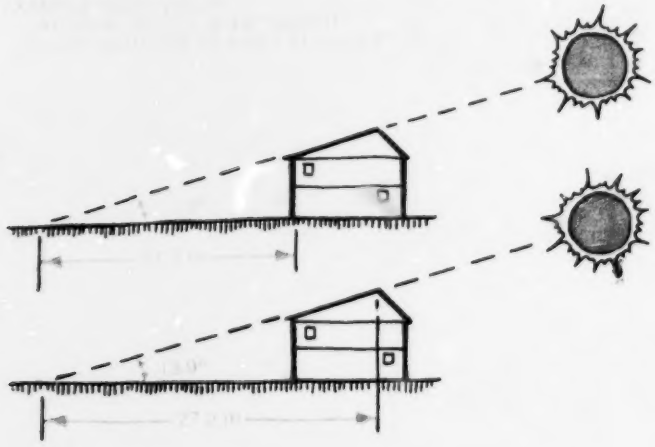


6. From each of the poles representing the roof edge, draw a line along the solar azimuth angle. This angle is shown in the table of hourly solar altitudes and azimuths. The length of this line is that distance measured in Step 3.

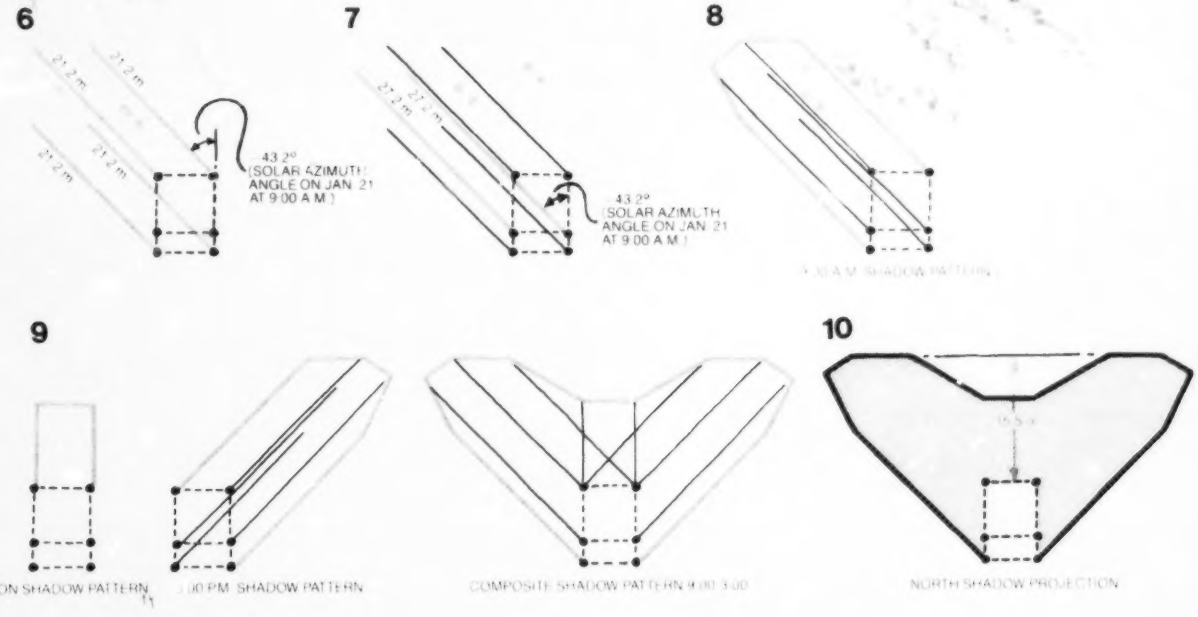
7. From each of the points representing the roof peak, draw a line along the solar azimuth angle. The length of this line is that distance measured in Step 4.

8. Connect the ends of the shadow lengths which approximate the shadow cast on the ground at a particular design hour.

9. Repeat Steps 2 through 8 for two other design hours of the day (in this example they are noon and 3:00 p.m.) to complete the daily shadow pattern.

10. In order to determine the minimum north-south separation distance between houses in a subdivision, measure the distance between the northernmost point of the house and the northernmost point of the shadow pattern (in plan view) to derive the north shadow projection.

    Also ensure that all dwellings, when they are sited, are outside the lateral shading limits of their neighbours by relating to the shadow projections determined by the azimuth angles of the first morning and last afternoon design hours.



Method B — Shadow length multiplier templates

Description

Shadow length multipliers are a series of ratios which, in conjunction with the height of the house, determine the shadow lengths at various times of the day, slope gradients and slope directions. Used together with shadow length multiplier templates, which indicate the angles at which the shadows fall, these provide a relatively simple method of constructing the shadow patterns of a house.

Templates are provided for January 21 covering the 9:00 a.m. to 3:00 p.m. design period only. The template method is derived from a concept developed by the Planning and Development Department of the Regional Municipality of Hamilton-Wentworth.

Three site terrain conditions are considered:

- 5 percent slope falling to the north
- Level ground
- 5 percent slope falling to the south

Information required

- Dimensions of the dwelling (in plan and elevation)
- Orientation of the dwelling
- Slope gradient and direction
- Day of the year
- Time of day (design hour)
- Latitude of the site
- Shadow Length Multiplier Template

Process

1. Designate to scale (in plan view) the house as a series of poles representing the heights of the roof edge at the corners of the building and the peak of the roof.
2. Select the Shadow Length Multiplier Template according to that template which most closely approximates the latitude of the particular site. These templates are found at the back of this document.
3. Overlay the template so that one of the poles of the house plan coincides with the centre point on the template, and the north point of the house plan is in line with the north point of the template.
4. In order to determine the directions of the shadows, transfer all three lines on the template radiating from the pole onto the plan by marking the directions at the edge of the template, and then connecting the marks with the pole.
5. It is now necessary to determine the actual lengths of the shadows. Multiply the height of the particular representative pole by the appropriate shadow length multiplier (which is specified in the table on the Shadow Length Multiplier Template) for each design hour. The product of the height of the pole and the multiplier yields the shadow length.

SLOPE DIRECTION: FALLING TO THE SOUTH
SLOPE GRADIENT: 5 PERCENT
LEVEL GROUND
SLOPE DIRECTION: FALLING TO THE NORTH
SLOPE GRADIENT: 5 PERCENT
9:00 A.M. (5.3 m) x (4.7) = 24.9 m
NOON (5.3 m) x (2.3) = 12.2 m
3:00 P.M. (5.3 m) x (4.7) = 24.9 m

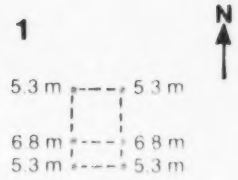
6. Mark off the shadow lengths (as determined in Step 5.) along the lines identified by the template (as determined in Step 4).
7. Repeat Steps 3 to 6 for the remaining representative poles. It is important to remember that the poles representing the roof peak are generally taller than those of the roof edge.
8. Connect the ends of the shadow lengths which approximate the shadow cast on the ground between 9:00 a.m. and 3:00 p.m.
9. In order to determine the minimum north-south separation distance between houses in a subdivision, measure the distance between the northernmost point of the house and the northernmost point of the shadow pattern (in plan view) to derive the north shadow projection.

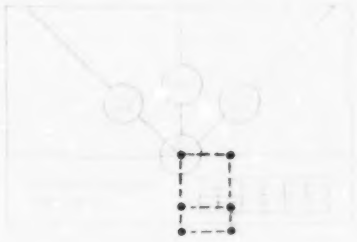
   Also ensure that all dwellings, when they are sited, are outside the lateral shading limits of their neighbours by relating to the shadow projections determined by the azimuth angles of the first morning and last afternoon design hours.

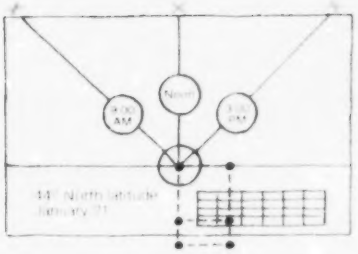
2,3

4

1





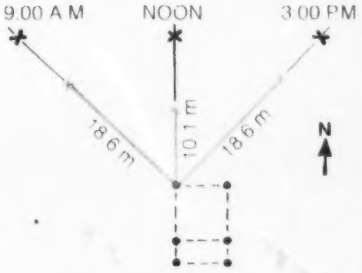


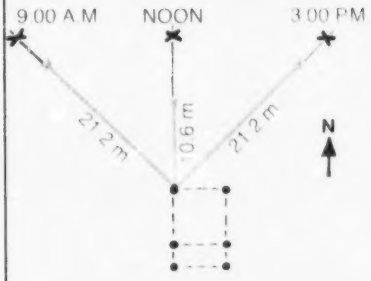
SLOPE DIRECTION:
FALLING TO THE SOUTH
SLOPE GRADIENT: 5 PERCENT

LEVEL GROUND

SLOPE DIRECTION
FALLING TO THE NORTH
SLOPE GRADIENT: 5 PERCENT

6





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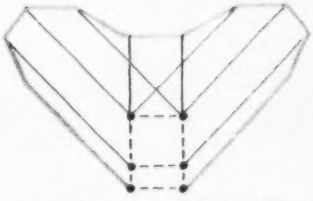
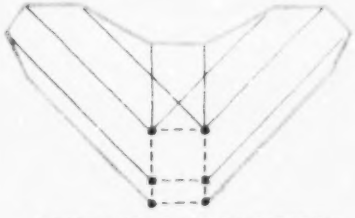


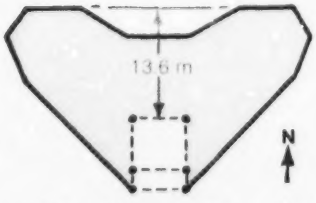
COMPOSITE SHADOW PATTERN
9:00 A.M.-3:00 P.M.



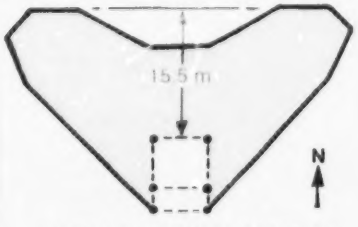
COMPOSITE SHADOW PATTERN
9:00 A.M.-3:00 P.M.

COMPOSITE SHADOW PATTERN
9:00 A.M.-3:00 P.M.

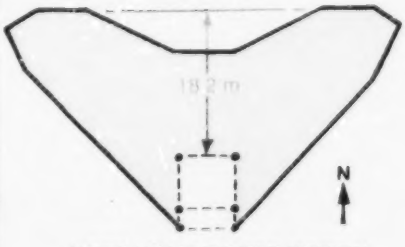
9



NORTH SHADOW PROJECTION



NORTH SHADOW PROJECTION



NORTH SHADOW PROJECTION

Method C — Shadow length formula

Description

This method requires the use of ratios and a mathematical formula. This formula is derived from the APA publication *Protecting Solar Access for Residential Development*.

Three site terrain conditions are considered in this example:

- 5 percent slope falling to the south
- level ground
- 5 percent slope falling to the north

Using this method affords the user a greater amount of precision and flexibility than provided by the other two methods previously described. The exact information related to the location of the sun in the sky based on the precise latitude of the site and preferred design times during the day, can be used to accurately determine shadow projections.

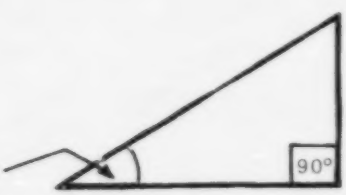
This method is based on two simple ratios that are concerned with the relationship of the sides and angles in a right-angled triangle: tangent (tan) and cosine (cos). The calculations are carried out using either the appropriate trigonometrical ratio tables or a pocket calculator with tangent and cosine functions.

For those shadow projections on level ground (which account for a large number of the development situations), there is nothing more to consider than the height of the dwelling and the altitude of the sun in order to determine shadow lengths. Therefore, a simple formula is used:

$$S = \frac{H}{\tan A}$$

where S is the shadow length
H is the height of the object casting the shadow
A is the altitude angle of the sun at a particular design hour as shown in the table of hourly solar altitudes and azimuths.

Determinants of shadow lengths on level ground.



TANGENT = $\frac{H}{S}$

COSINE = $\frac{S}{X}$

In the case of shadow lengths on a sloped surface, the calculation becomes more complex. The difference between calculating for shadows cast on level ground and shadows on sloping terrain is the result of two additional considerations:

- The relationship between the slope direction of the land and the altitude of the sun.

  As the land rises in the same direction as the direction of the sun's rays (south slopes), the shadow becomes shorter. When the land falls away from the direction of the sun's rays (north slopes), the shadow becomes longer.

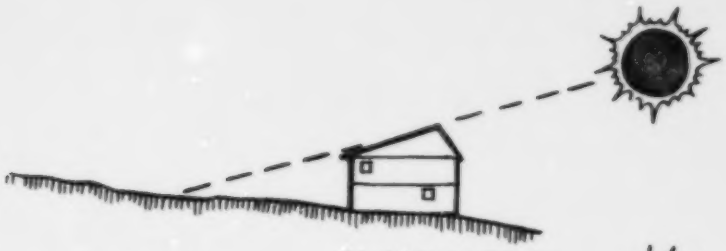
- The relationship between the sun's azimuth angle and the direction of the slope of the land.

  The sun's rays do not generally fall in the exact same direction as the slope of the land. In other words, the azimuth of the sun and the direction of the land's slope are not usually equal. Compensation must be made for this difference.

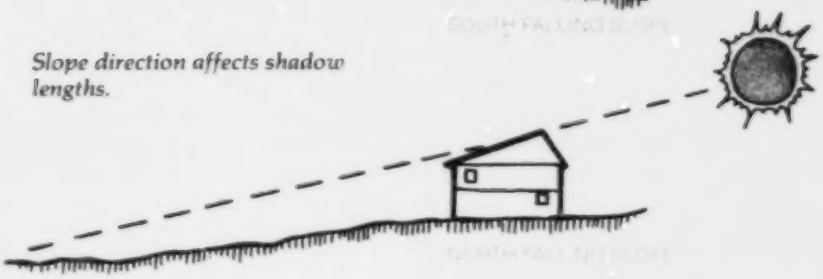
The equation which considers these two factors is:

$$S = \frac{H}{\tan A + B\,[\cos (C - D)]}$$

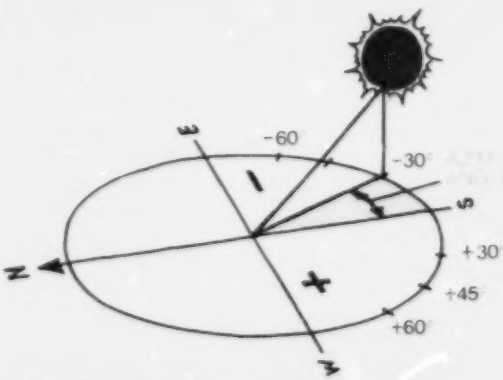
where S is the shadow length
H is the height of the object casting the shadow
A is the solar altitude angle
B is the percentage slope of the land
C is the azimuth angle of the sun as shown in the table of hourly solar altitudes and azimuths
D is the direction of the slope of the land.



Slope direction affects shadow lengths.



Determinants of shadow lengths on a sloping surface.



Azimuth angle of the sun to the object casting the shadow.

Slope direction of the land from the object casting the shadow.

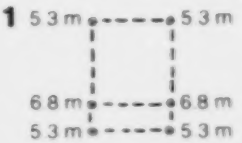
Information Required

- Dimensions of the dwelling (in plan and elevation)
- Orientation of the dwelling
- Slope gradient and direction
- Day of the year
- Time of day (design hour)
- Latitude of the site
- Solar azimuth
- Solar altitude

Process

1. Designate to scale (in plan view) the house as a series of poles representing the heights of the roof edge at the corners of the building and the peak of the roof.

1



2. (a) For the situation where the house is on level ground, use the following equation to determine the shadow lengths of each of the representative poles at a particular design hour:

$$\text{shadow length} = \frac{H}{\tan A}$$

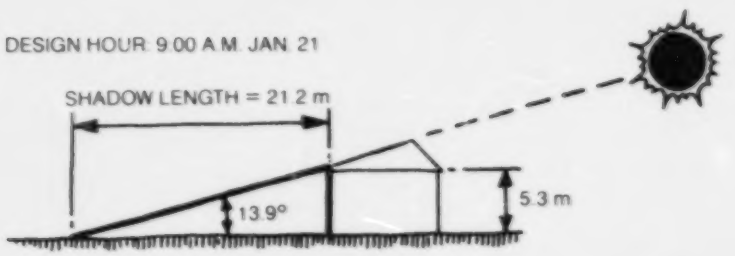
where H is the height of the pole
A is the solar altitude angle at a particular design hour.

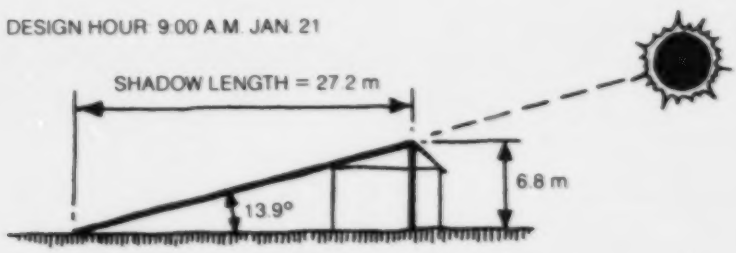
2. (b) For the situation where the house is on a sloping (greater than 0 percent) surface, use the following equation to determine the shadow lengths of each of the representative poles at a particular design hour:

$$\text{shadow length} = \frac{H}{\tan A + B[\cos (C - D)]}$$

where H is the height of the representative pole casting a shadow
A is the solar altitude angle
B is the percentage slope of the land
C is the azimuth angle of the sun as shown in the table of hourly solar altitudes and azimuths
D is the direction of the slope of the land

2a

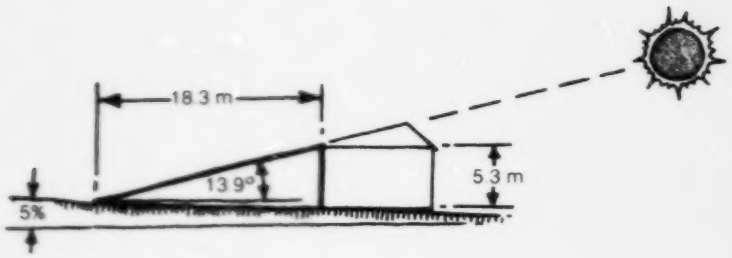




2b

5% south falling slope

SLOPE DIRECTION: FALLING TO THE SOUTH
SLOPE GRADIENT: 5 PERCENT



the shadow length of representative pole 5.3 m high =

$$\frac{H}{\tan A + B\,[\cos\,(C - D)]}$$

$$= \frac{5.3\ m}{\tan 13.9° + .05\,[\cos\,(-43.2° - 0°)]}$$

$$= \frac{5.3\ m}{.25 + .05\,[-0.73]}$$

$$= \frac{5.3\ m}{.25 + [-0.04]}$$

$$= \frac{5.3\ m}{.29}$$

$$= 18.3\ m$$

the shadow length of representative pole 6.8 m high =

$$\frac{H}{\tan A + B\,[\cos\,(C - D)]}$$

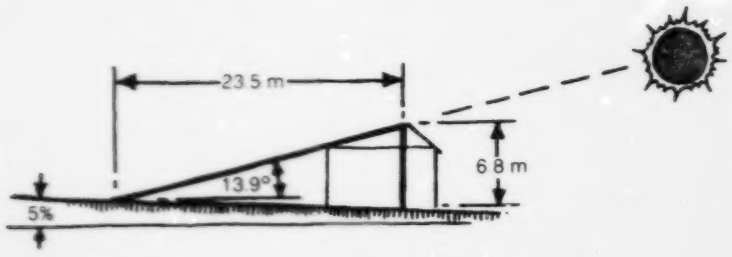
$$= \frac{6.8\ m}{\tan 13.9° + .05\,[\cos\,(-43.2° - 0°)]}$$

$$= \frac{6.8\ m}{.25 + .05\,[-0.73]}$$

$$= \frac{6.8\ m}{.25 + [-0.04]}$$

$$= \frac{6.8\ m}{.29}$$

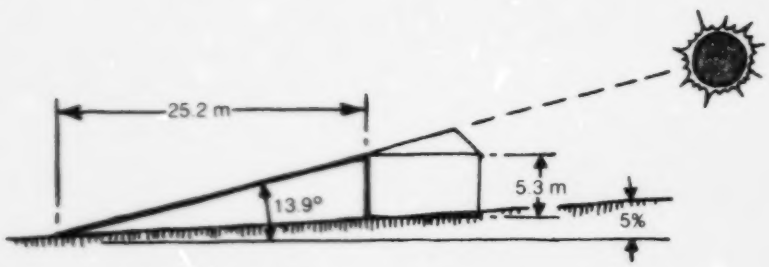
$$= 23.5\ m$$



2b

5% north falling slope

SLOPE DIRECTION: FALLING TO THE NORTH
SLOPE GRADIENT: 5 PERCENT



the shadow length at representative pole 5.3 m high =

$$\frac{H}{\tan A + B\,[\cos\,(C - D)]}$$

$$= \frac{5.3\ m}{\tan 13.9° + .05\,[\cos\,(-43.2° - 180°)]}$$

$$= \frac{5.3\ m}{.25 + .05\,[-0.73]}$$

$$= \frac{5.3\ m}{.25 + [-0.04]}$$

$$= \frac{5.3\ m}{.21}$$

$$= 25.2\ m$$

the shadow length of representative pole 6.8 m high =

$$\frac{H}{\tan A + B\,[\cos\,(C - D)]}$$

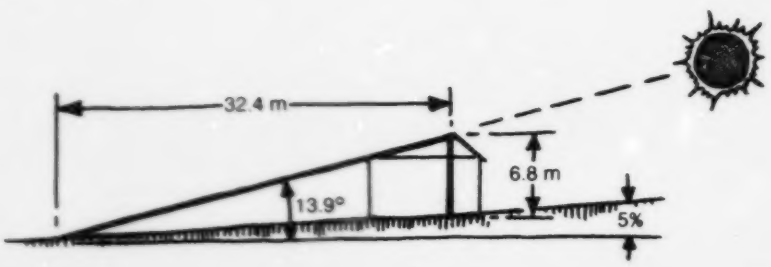
$$= \frac{6.8\ m}{\tan 13.9° + .05\,[\cos\,(-43.2° - 180°)]}$$

$$= \frac{6.8\ m}{.25 + .05\,[-0.73]}$$

$$= \frac{6.8\ m}{.25 + [-0.04]}$$

$$= \frac{6.8\ m}{.21}$$

$$= 32.4\ m$$

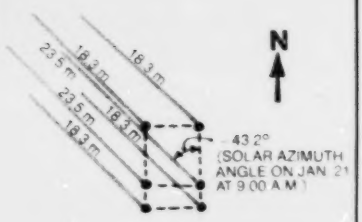


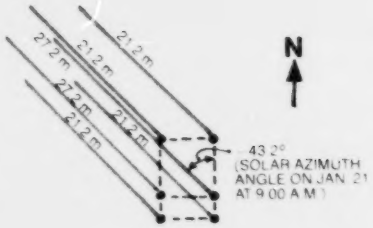
SLOPE DIRECTION
FALLING TO THE SOUTH
SLOPE GRADIENT: 5 PERCENT

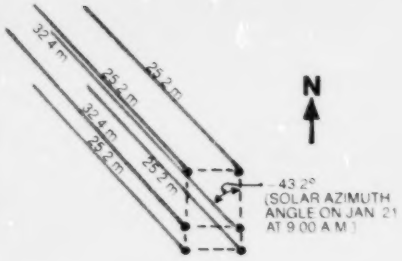
LEVEL GROUND

SLOPE DIRECTION
FALLING TO THE NORTH
SLOPE GRADIENT: 5 PERCENT

3







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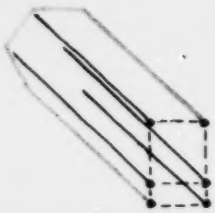
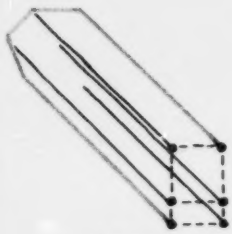
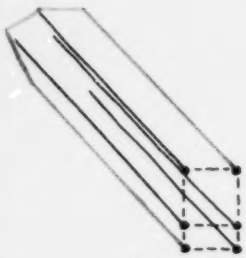


9:00 A.M. SHADOW PATTERN

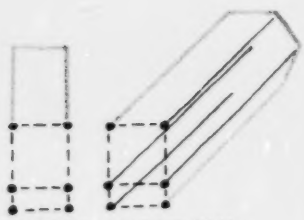


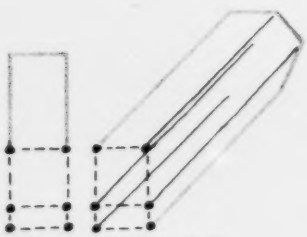
9:00 A.M. SHADOW PATTERN

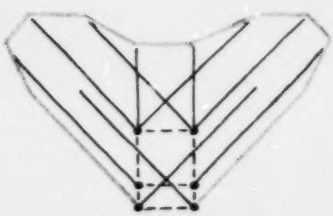


9:00 A.M. SHADOW PATTERN

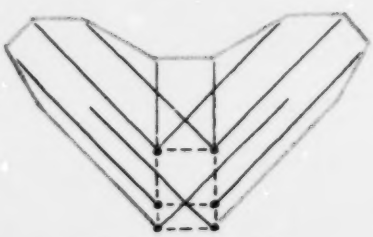
5



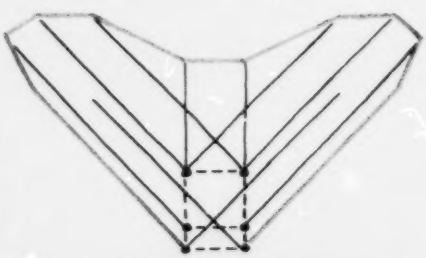




COMPOSITE SHADOW PATTERN
9:00-3:00

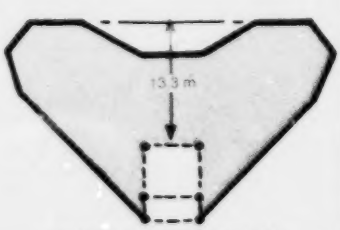


COMPOSITE SHADOW PATTERN
9:00-3:00

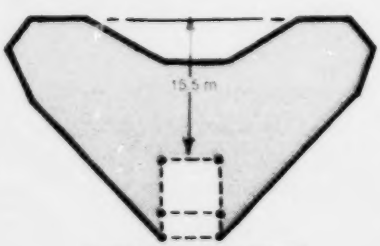


COMPOSITE SHADOW PATTERN
9:00-3:00

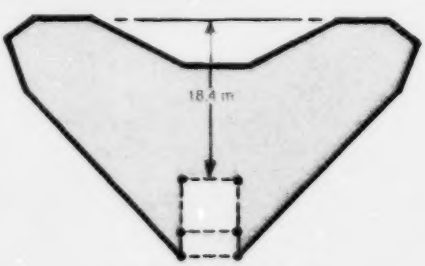
6



NORTH SHADOW PROJECTION



NORTH SHADOW PROJECTION



NORTH SHADOW PROJECTION

3. From each of the poles representing the different heights of the house, draw a line along the solar azimuth angle.

   The length of this line is that distance determined in Step 2.

4. Connect the ends of the shadow lengths which approximate the shadow cast on the ground at a particular hour.

5. Repeat Steps 2 to 4 for two other design hours of the day to complete the daily shadow pattern.

6. In order to determine the minimum north-south separatio ' distance between houses in a subdivision, measure the distance between the northernmost point of the house and the northernmost point of the shadow pattern to derive the north shadow projection.

   Also ensure that all dwellings, when they are sited, are outside the lateral shading limits of their neighbours by relating to the shadow projections determined by the azimuth angles of the first morning and last afternoon design hours.

Comparison of methods

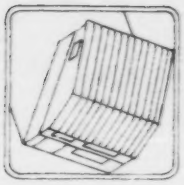
The following table describes the flexibility of the shadow calculation methods in dealing with the two variables:

- Slope direction and gradient of the land
- Design period for determining shadow patterns

In general, the Shadow Length Formula is the most flexible and accurate of the methods. For most purposes, however, the degree of accuracy provided by the other two methods is sufficient.

	Slope direction and gradient	**Design period**
Method A (shadows from geometry)	should not be applied to slope situations	applies to any design period
Method B (shadow length multiplier tables)	applies to eight specific slope directions and up to 5 percent gradient	9:00 a.m. to 3:00 p.m. design period only
Method C (shadow length formula)	applies to all slope directions and gradients	applies to any design period

Conclusion



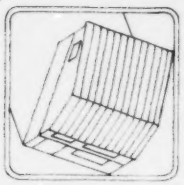
Using energy from the sun to heat homes helps achieve this Province's goal of reduced dependency on fossil fuels. In order to make use of this solar heat, houses must be provided with unobstructed access for particular times of the day and months of the year.

Determining shadow patterns using the methods described in this document requires a planner to put some extra thought into the subdivision design. Depending on the number and types of dwellings being built, a certain amount of calculation is necessary.

However, once the methods are understood and the appropriate method selected, the calculation of shadow patterns is likely to be easily absorbed into the subdivision process.

Application of shadow patterns is to be further explored in the parts of the Handbook dealing with landscaping, subdivision examples, and medium density ground-related housing forms.

Glossary of terms



Design hour:

a particular hour within the design period.

Design period:

a time span of the day during which it is considered important for a building to have unobstructed access to the sun on its south facing wall.

Orientation:

the location of the wall of a dwelling with the greatest amount of window area in relation to the points of the compass.

Passive solar energy:

energy from the sun which is utilized without employing technical apparatuses and conversion processes.

Percentage slope:

the result (expressed as a percentage) of dividing the difference in height between the low and high points of a slope by the distance between these two points as measured on a map.

Shadow length:

the distance and direction of shadow cast by an object at any given time.

Shadow pattern:

composite of the shadow lengths of the representative poles at any given time (or times).

Shadow projection (lateral):

the western and eastern angular extensions of the shadow cast by an object determined by the sun's azimuth.

Shadow projection (north):

the distance measured along a north-south axis between the shadow casting object and the northernmost distance of the shadow pattern.

Solar access:

allowing sunlight to penetrate within a building, while the effects of potential obstructions, such as shadows cast by buildings and trees, are eliminated by the proper siting of these elements.

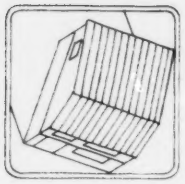
Solar altitude:

angle used to specify the height of the sun above the horizon at any given time.

Solar azimuth:

angle used to specify the bearing of the sun at any given time.

List of publications for further reference



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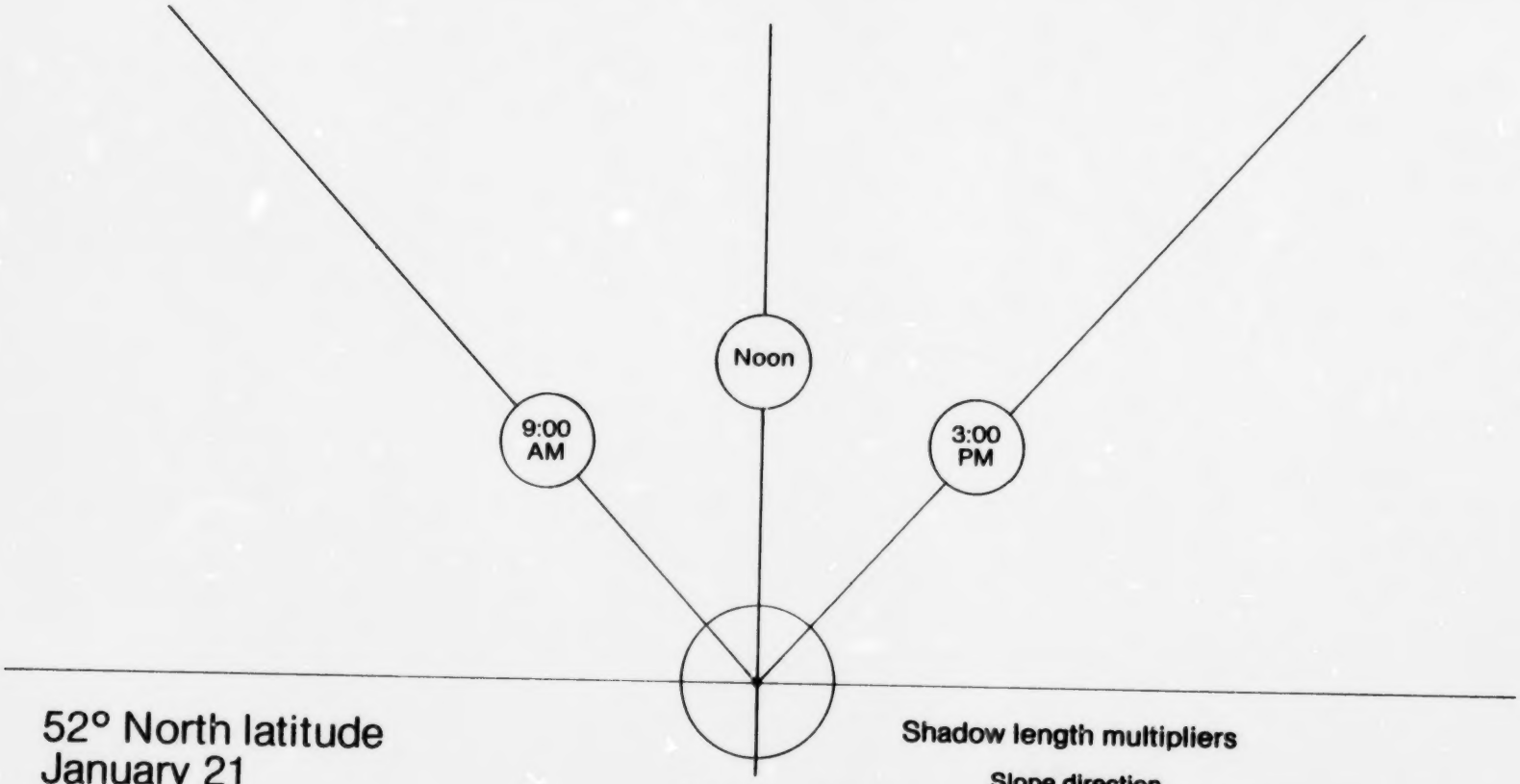


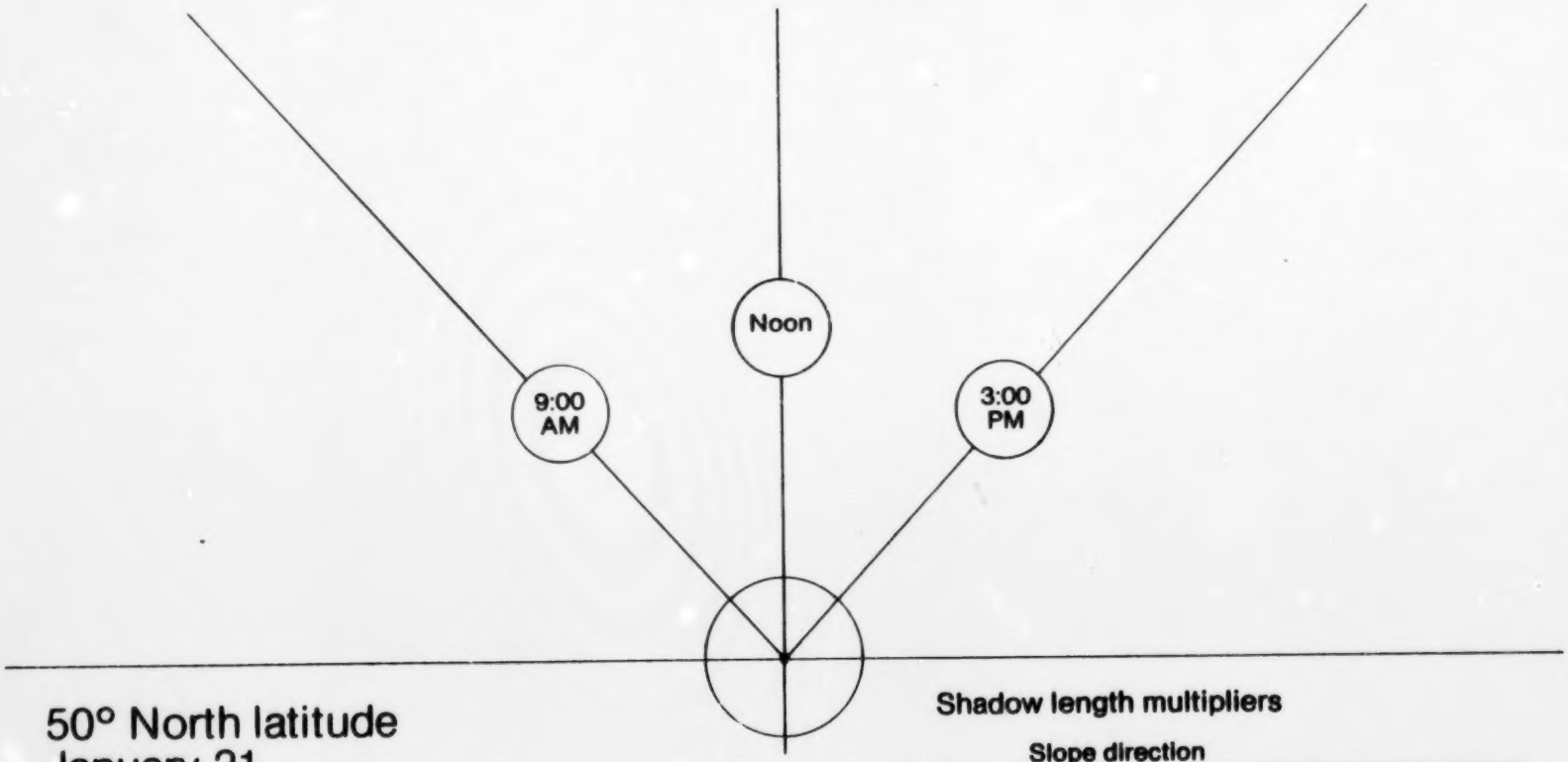
52° North latitude
January 21

Shadow length = Shadow length multiplier X Height of representative pole

Shadow length multipliers

Slope direction

Slope %	NORTH			NORTHEAST			EAST			SOUTHEAST			SOUTH			SOUTHWEST			WEST			NORTHWEST		
	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm
0%	7.1	3.1	7.1	7.1	3.1	7.1	7.1	3.1	7.1	7.1	3.1	7.1	7.1	3.1	7.1	7.1	3.1	7.1	7.1	3.1	7.1	7.1	3.1	7.1
1%	7.5	3.2	7.5	7.1	3.2	7.7	6.8	3.1	7.5	6.7	3.0	7.1	6.8	3.0	6.8	7.1	3.0	6.7	7.5	3.1	6.8	7.7	3.2	7.1
2%	8.0	3.3	8.0	7.2	3.2	8.3	6.5	3.1	7.9	6.2	3.0	7.1	6.4	2.9	6.4	7.1	3.0	6.2	7.9	3.1	6.5	8.3	3.2	7.2
3%	8.5	3.4	8.5	7.2	3.3	9.1	6.2	3.1	8.3	5.9	2.9	7.0	6.2	2.8	6.2	7.0	2.9	5.9	8.3	3.1	6.2	9.1	3.3	7.2
4%	9.0	3.5	9.0	7.2	3.4	10.0	6.0	3.1	8.8	5.5	2.8	7.0	5.9	2.7	5.9	7.0	2.8	5.5	8.8	3.1	6.0	10.0	3.4	7.2
5%	9.7	3.6	9.7	7.2	3.5	11.0	5.7	3.1	9.4	5.3	2.8	7.0	5.6	2.7	5.6	7.0	2.8	5.3	9.4	3.1	5.7	11.0	3.5	7.2



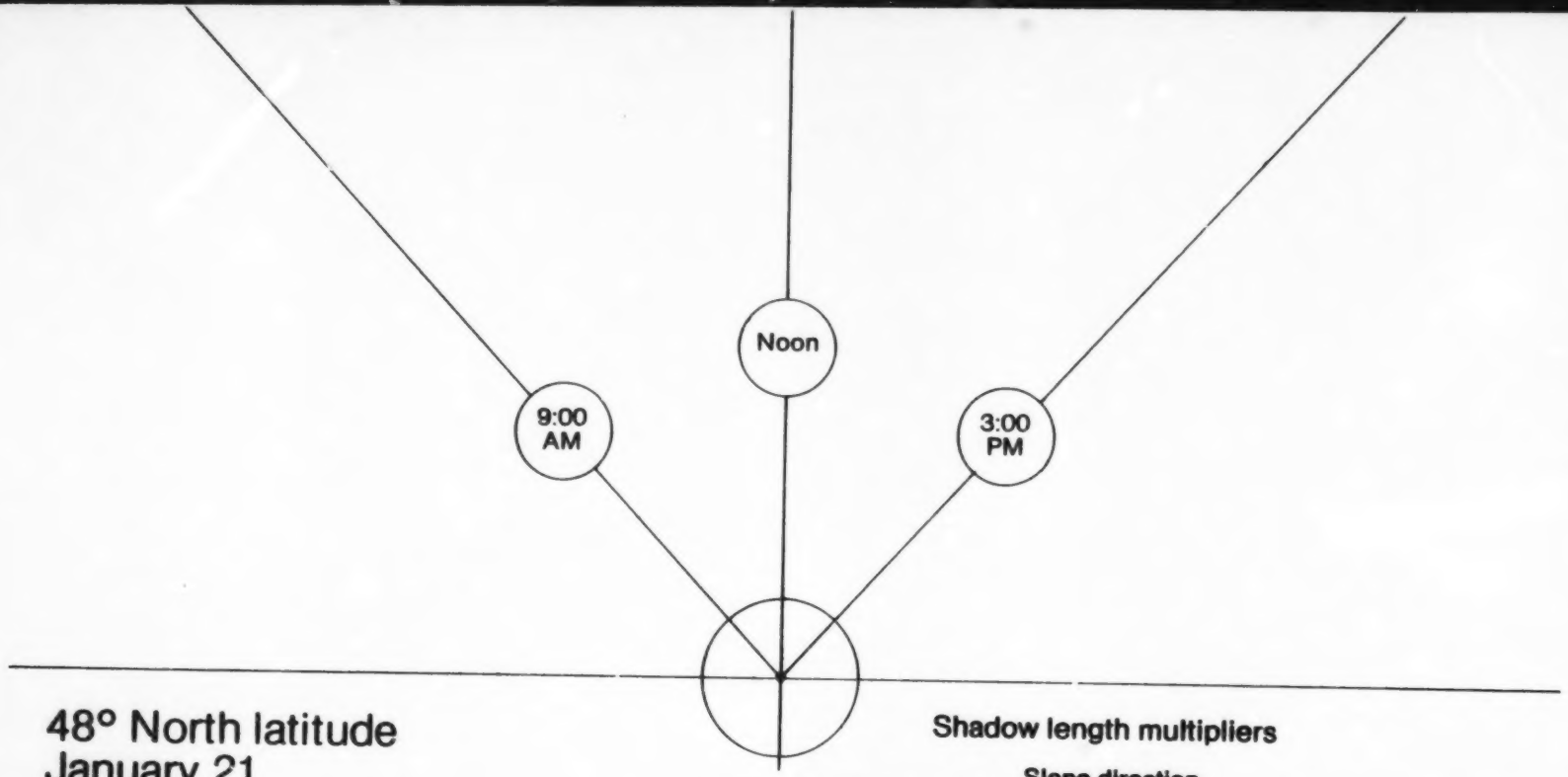
50° North latitude
January 21

Shadow length = Shadow length multiplier X Height of representative pole

Shadow length multipliers

Slope direction

Slope %	NORTH			NORTHEAST			EAST			SOUTHEAST			SOUTH			SOUTHWEST			WEST			NORTHWEST		
	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm
0%	6.0	2.8	6.0	6.0	2.8	6.0	6.0	2.8	6.0	6.0	2.8	6.0	6.0	2.8	6.0	6.0	2.8	6.0	6.0	2.8	6.0	6.0	2.8	6.0
1%	6.3	2.8	6.3	6.0	2.8	6.4	5.8	2.8	6.2	5.6	2.7	6.0	5.7	2.7	5.7	6.0	2.7	5.6	6.2	2.8	5.8	6.4	2.8	6.0
2%	6.6	2.9	6.6	6.0	2.9	6.8	5.5	2.8	6.5	5.3	2.7	5.9	5.5	2.6	5.5	5.9	2.7	5.3	6.5	2.8	5.5	6.8	2.9	6.0
3%	6.9	3.0	6.9	6.0	2.9	7.3	5.3	2.8	6.8	5.1	2.6	5.9	5.3	2.5	5.3	5.9	2.6	5.1	6.8	2.8	5.3	7.3	2.9	6.0
4%	7.3	3.1	7.3	6.1	3.0	7.9	5.2	2.8	7.1	4.8	2.6	5.9	5.1	2.5	5.1	5.9	2.6	4.8	7.1	2.8	5.2	7.9	3.0	6.1
5%	7.7	3.2	7.7	6.1	3.0	8.5	5.0	2.8	7.5	4.6	2.5	5.9	4.9	2.4	4.9	5.9	2.5	4.6	7.5	2.8	5.0	8.5	3.0	6.1



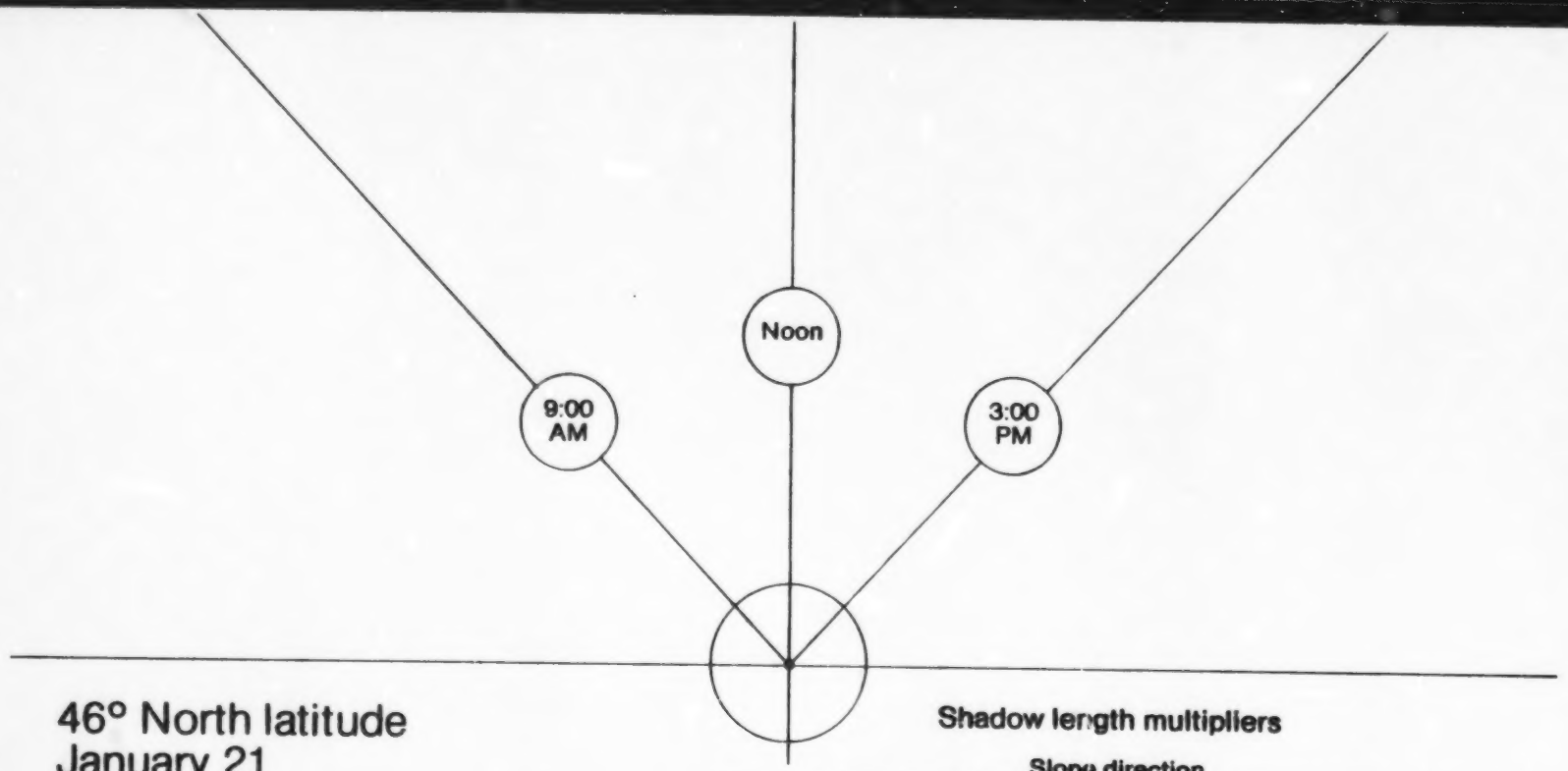
48° North latitude
January 21

Shadow length multipliers

Slope direction

Slope %	NORTH			NORTHEAST			EAST			SOUTHEAST			SOUTH			SOUTHWEST			WEST			NORTHWEST		
	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm
0%	5.2	2.5	5.2	5.2	2.5	5.2	5.2	2.5	5.2	5.2	2.5	5.2	5.2	2.5	5.2	5.2	2.5	5.2	5.2	2.5	5.2	5.2	2.5	5.2
1%	5.4	2.5	5.4	5.2	2.5	5.5	5.0	2.5	5.4	4.9	2.4	5.2	5.0	2.4	5.0	5.2	2.4	4.9	5.4	2.5	5.0	5.5	2.5	5.2
2%	5.6	2.6	5.6	5.2	2.6	5.8	4.9	2.5	5.6	4.7	2.4	5.2	4.8	2.4	4.8	5.2	2.4	4.7	5.6	2.5	4.9	5.8	2.6	5.2
3%	5.9	2.7	5.9	5.2	2.6	6.2	4.7	2.5	5.8	4.5	2.4	5.2	4.7	2.3	4.7	5.2	2.4	4.5	5.8	2.5	4.7	6.2	2.6	5.2
4%	6.1	2.8	6.1	5.2	2.7	6.6	4.6	2.5	6.0	4.3	2.3	5.2	4.5	2.3	4.5	5.2	2.3	4.3	6.0	2.5	4.6	6.6	2.7	5.2
5%	6.4	2.8	6.4	5.3	2.7	7.0	4.4	2.5	6.3	4.1	2.3	5.1	4.4	2.2	4.4	5.1	2.3	4.1	6.3	2.5	4.4	7.0	2.7	5.3

Shadow length = Shadow length multiplier X Height of representative pole



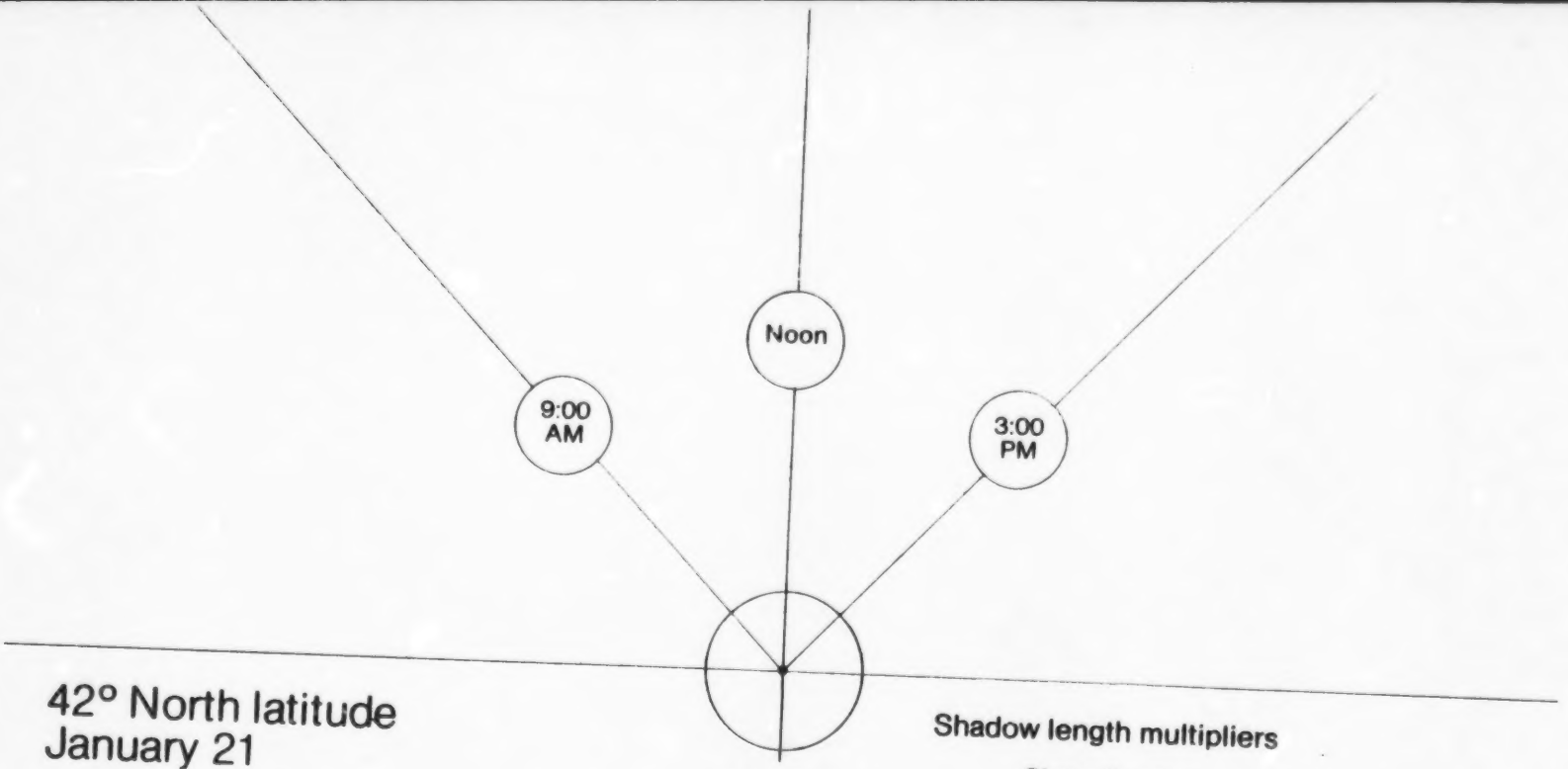
46° North latitude
January 21

Shadow length multipliers

Slope direction

Slope %	NORTH			NORTHEAST			EAST			SOUTHEAST			SOUTH			SOUTHWEST			WEST			NORTHWEST		
	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm
0%	4.5	2.3	4.5	4.5	2.3	4.5	4.5	2.3	4.5	4.5	2.3	4.5	4.5	2.3	4.5	4.5	2.3	4.5	4.5	2.3	4.5	4.5	2.3	4.5
1%	4.7	2.3	4.7	4.5	2.3	4.7	4.4	2.3	4.7	4.3	2.2	4.5	4.4	2.2	4.4	4.5	2.2	4.3	4.7	2.3	4.4	4.7	2.3	4.5
2%	4.8	2.4	4.8	4.5	2.3	5.0	4.3	2.3	4.8	4.1	2.2	4.5	4.2	2.2	4.2	4.5	2.2	4.1	4.8	2.3	4.3	5.0	2.3	4.5
3%	5.0	2.4	5.0	4.5	2.4	5.2	4.1	2.3	5.0	4.0	2.1	4.5	4.1	2.1	4.1	4.5	2.1	4.0	5.0	2.3	4.1	5.2	2.4	4.5
4%	5.2	2.5	5.2	4.5	2.4	5.5	4.0	2.3	5.1	3.8	2.1	4.5	4.0	2.1	4.0	4.5	2.1	3.8	5.1	2.3	4.0	5.5	2.4	4.5
5%	5.4	2.5	5.4	4.6	2.4	5.8	3.9	2.3	5.3	3.7	2.1	4.5	3.9	2.0	3.9	4.5	2.1	3.7	5.3	2.3	3.9	5.8	2.4	4.6

Shadow length = Shadow length multiplier X Height of representative pole



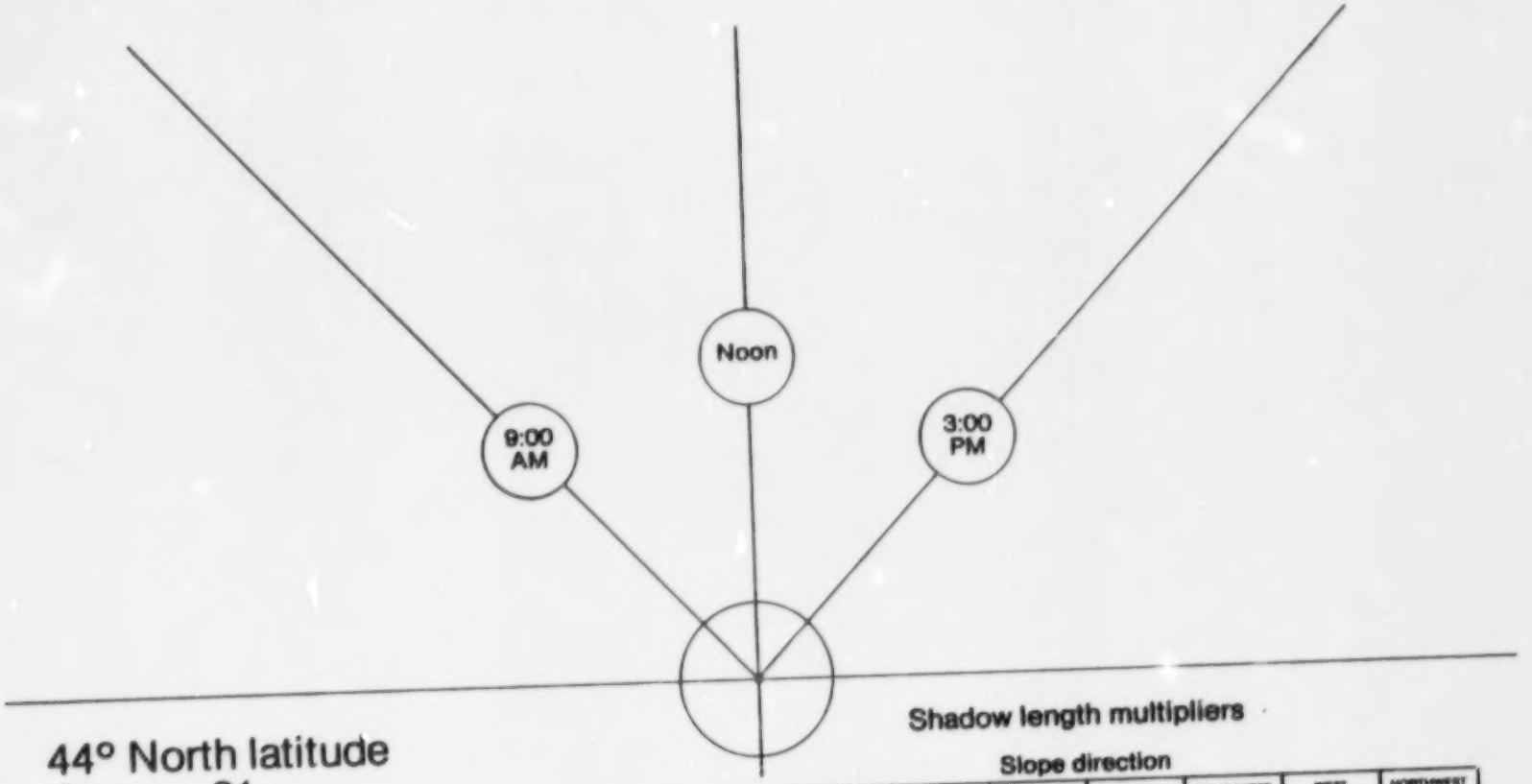
42° North latitude
January 21

Shadow length = Shadow length multiplier X Height of representative pole

Shadow length multipliers

Slope direction

Slope %	NORTH			NORTHEAST			EAST			SOUTHEAST			SOUTH			SOUTHWEST			WEST			NORTHWEST		
	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm
0%	37	19	37	37	19	37	37	19	37	37	19	37	37	19	37	37	19	37	37	19	37	37	19	37
1%	38	19	38	37	19	38	36	19	38	35	19	37	36	19	36	37	19	35	38	19	36	38	19	37
2%	39	20	39	37	19	39	35	19	39	34	18	36	35	18	35	36	18	34	39	19	35	39	19	37
3%	40	20	40	37	20	41	34	19	40	33	18	36	34	18	34	36	18	33	40	19	34	41	20	37
4%	41	20	41	37	20	43	33	19	41	32	18	36	33	18	33	36	18	32	41	19	33	43	20	37
5%	42	21	42	37	20	45	32	19	42	31	18	36	32	17	32	36	18	31	42	19	32	45	20	37



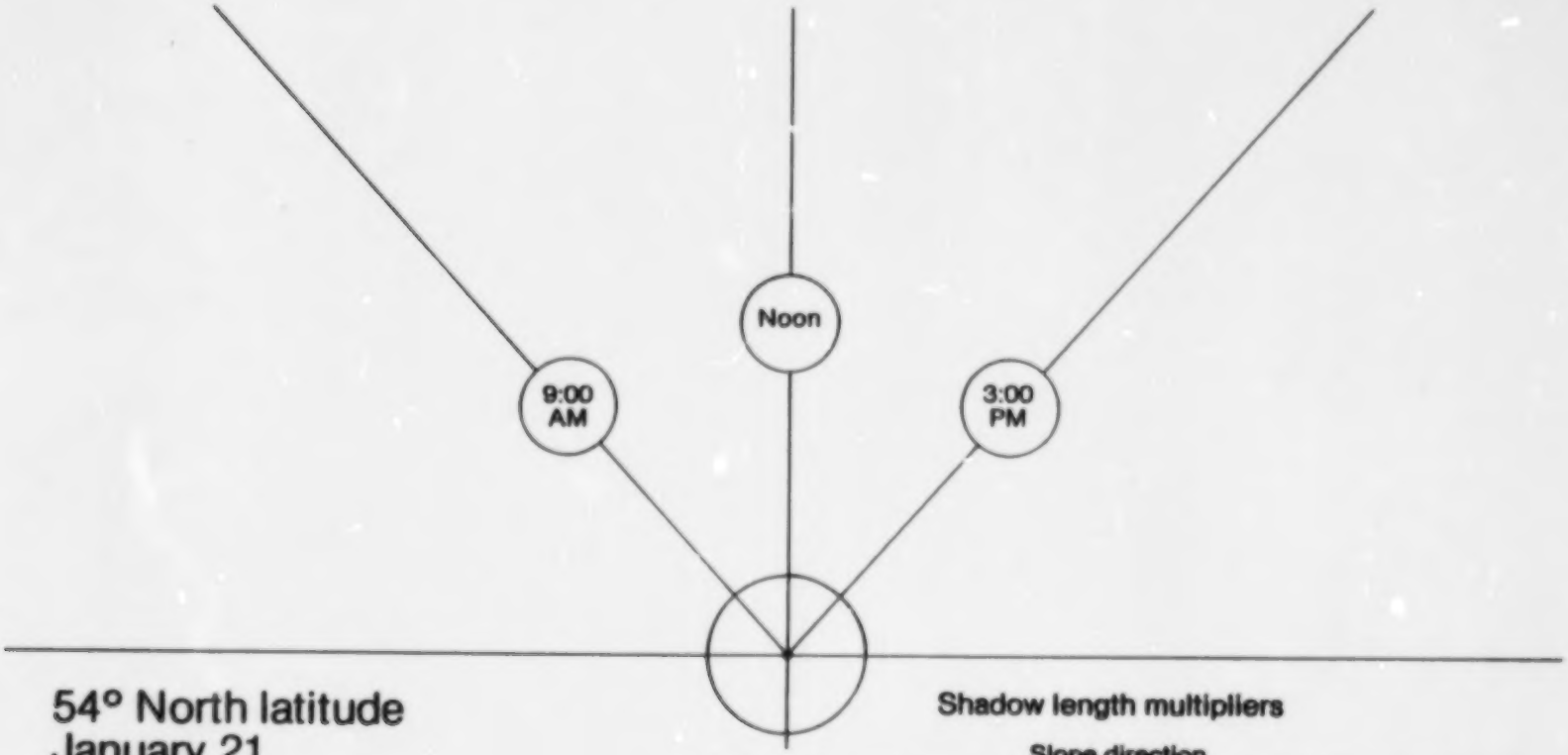
44° North latitude
January 21

Shadow length = Shadow length multiplier X Height of representative pole

Shadow length multipliers

Slope direction

Slope %	NORTH			NORTHEAST			EAST			SOUTHEAST			SOUTH			SOUTHWEST			WEST			NORTHWEST		
	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm
0%	40	21	40	40	21	40	40	21	40	40	21	40	40	21	40	40	21	40	40	21	40	40	21	40
1%	42	21	42	41	21	42	39	21	42	39	20	40	39	20	39	40	20	39	42	21	39	42	21	41
2%	43	21	43	41	21	44	38	21	43	37	20	40	38	20	38	40	20	37	43	21	38	44	21	41
3%	44	22	44	41	21	46	37	21	44	36	20	40	37	19	37	40	20	36	44	21	37	46	21	41
4%	46	22	46	41	22	48	36	21	45	35	19	40	36	19	36	40	19	35	45	21	36	48	22	41
5%	47	23	47	41	22	51	36	21	47	34	19	40	35	19	35	40	19	34	47	21	36	51	22	41



54° North latitude
January 21

Shadow length = **Shadow length multiplier** X **Height of representative pole**

Shadow length multipliers

Slope direction

Slope %	NORTH			NORTHEAST			EAST			SOUTHEAST			SOUTH			SOUTHWEST			WEST			NORTHWEST		
	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm	9am	noon	3pm
0%	8.6	3.5	8.6	8.6	3.5	8.6	8.6	3.5	8.6	8.6	3.5	8.6	8.6	3.5	8.6	8.6	3.5	8.6	8.6	3.5	8.6	8.6	3.5	8.6
1%	9.2	3.6	9.2	9.7	3.6	9.5	8.2	3.5	9.2	8.0	3.4	8.8	8.1	3.4	8.1	8.6	3.4	8.0	9.2	3.5	8.2	9.5	3.6	8.7
2%	9.9	3.8	9.9	9.7	3.7	10.5	7.8	3.5	9.8	7.4	3.3	8.0	7.7	3.3	7.7	8.6	3.3	7.4	9.8	3.5	7.8	10.5	3.7	8.7
3%	10.7	3.9	10.7	9.8	3.8	11.7	7.4	3.5	10.5	6.9	3.3	8.5	7.3	3.2	7.3	8.5	3.3	6.9	10.5	3.5	7.4	11.7	3.8	8.8
4%	11.6	4.1	11.6	9.8	3.9	13.2	7.0	3.5	11.2	6.4	3.2	8.5	6.9	3.1	6.9	8.5	3.2	6.4	11.2	3.5	7.0	13.2	3.9	8.9
5%	12.7	4.2	12.7	9.8	4.0	15.2	6.7	3.5	12.2	6.0	3.1	8.5	6.5	3.0	6.5	8.5	3.1	6.0	12.2	3.5	6.7	15.2	4.0	8.9

POOR

QUALITY

ORIGINAL